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CONTRAST DETECTION AND NEAR-THRESHOLD DISCRIMINATION IN HUMAN VISION

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Abstract—Forced-choice psychometric functions were determined for the detection of sinewave gratings and contrast discrimination of near-threshold gratings at spatial frequencies of 0.5, 2 and 8 c/deg. Detection psychometric functions all had the same S-shaped form. Discrimination functions were almost linear except at the upper end. Both sets of data can be described well by a detection model with a positively accelerating relation between contrast and mean decision variable and a differencing decision rule. Results of a paired comparisons experiment were consistent with the model and indicate that decision variable variance is nearly constant over the range of contrasts used in these experiments. The implications of these results for several models of contrast detection and discrimination are considered.

INTRODUCTION

This study is concerned with the detection of sine-wave luminance patterns and the discrimination of near-threshold patterns which differ only in contrast. Nachmias and Sansbury (1974) showed that forced-choice psychometric functions for contrast detection and contrast discrimination are markedly different in form and that discrimination thresholds near the detection threshold are substantially lower than detection thresholds. This is sometimes referred to as the pedestal effect or facilitation. Nachmias and Sansbury also showed that both functions are reasonably well described by a simple detection model employing a differencing decision rule. In the context of this model the results imply that the relation between contrast and the magnitude of the decision variable is characterized by a positively accelerating nonlinearity.* Other results consistent with this hypothesis have been obtained by Stromeyer and Klein (1974), Sansbury (1977) and Van Meeteren (1978).

The present study examines contrast detection and discrimination more thoroughly, both empirically and theoretically. We used three spatial frequencies and determined eight points on each psychometric function. In addition to the detection and discrimination paradigms, we also employed a paired-comparison procedure. Each of the paradigms involved a two-alternative temporal forced-choice paradigm. In each case the task was the same. Observers indicated which of the two intervals contained the higher contrast. The paradigms differed only in the selection of the contrasts to be discriminated. In the *detection paradigm* one interval contained zero contrast and the other contained one of a set of eight contrasts. In the *discrimination paradigm* one interval contained a constant, near-threshold contrast (pedestal) and the other contained this contrast plus an increment. In the *paired comparisons paradigm* a pair of unequal contrasts was selected randomly from a set of four near-threshold values, including zero. The results of these experiments are compared with the expectations inferred from several models of contrast detection and discrimination.

* This function is sometimes referred to as the overall transducer function (Nachmias, 1972). This term is avoided here because it leads to confusion between this function and theoretical interpretations of it, specifically, the nonlinear transducer models.

METHOD

Apparatus

Vertical sine-wave gratings were generated on a CRT display by Z-axis modulation using the method of Campbell and Green (1965). The display, designed and constructed at the Physiological Laboratory, Cambridge, had a P31 phosphor. The constant mean luminance was 170 cd/m^2. In the experiments to be reported, all stimuli were within the range in which display contrast was linearly related to Z-axis voltage.

Luminance modulation was horizontally restricted to a region symmetrical about the center of the screen. The remainder of the screen was maintained at the constant mean luminance level without modulation. This mode of presentation was produced by passing the Z-axis signal through an electronic switch. The onset and period of switch closure were controlled by logic pulses that corresponded to that part of the display sweep for which luminance modulation was desired. The Z-axis sinewave input was derived from a function generator. Input voltage and duration were controlled by a DEC PDP-8 computer. The circuitry is discussed in more detail in Legge (1979). During each observation interval, the computer generated a signal that was converted by a D/A converter into a DC voltage proportional to the desired contrast. The output of the function generator was multiplied by this voltage.

Procedure

Observers viewed the display binocularly, with natural pupils at a distance of 114 cm. The screen subtended 10° horizontally and 6° vertically. It was surrounded by a white surface of approximately the same luminance. Stimulus gratings were 6° × 6° and were centered in the display. Observers fixated a mark in the center of the display. All gratings were presented in cosine phase with respect to this mark.

A two-alternative temporal forced-choice paradigm was used (Green and Swets, 1966). Each trial consisted of two, 100 msec observation intervals, marked by tones, and separated by 600 msec. A signal grating was always presented in one and only one of the two intervals. The other interval contained either no grating or a grating of lower contrast. The signal was presented randomly in the first or second interval with equal probability. The observer indicated which interval contained the signal by pressing one of two keys. A correct choice was followed by another tone. To minimize the observer's uncertainty about stimulus parameters, a 100 msec glimpse of the stimulus with a contrast approx. 2.4 times threshold was presented just prior to each trial.

Forced-choice psychometric functions were determined in the following way. A forced-choice staircase procedure (Legge, 1979) was used to estimate the contrast that yielded 75% correct responses. Then eight contrast values were selected to use in determining the psychometric function. These included the contrast yielding about 75% correct plus seven other values, four below it having contrasts of 25, 50, 67 and 83% of this value, and three above it with contrasts of 117, 133 and 150% of this value. These were presented with equal probability in a strictly random order. The observer initiated each trial when ready. Typically 3–5 sec intervened between trials. A block of trials on a single condition usually consisted of 450 trials. With the subject being allowed a brief break after each 150 trials, a block took about 45 min. The psychometric functions in Fig. 1 are each based on four blocks or 1800 trials. Each point is based on approximately 200 trials.

There were three paradigms used in the experiment: detection, near-threshold discrimination, and paired comparisons. In the detection paradigm a signal grating was presented randomly in one of the two observation intervals. No grating was presented in the other interval, the field remaining homogeneous and the luminance constant. In the near-threshold discrimination paradigm a background grating of constant contrast (pedestal) corresponding approximately to the detection threshold (75% correct) was presented in both observation intervals of every trial. The signal grating was added to this pedestal in one of the two observation intervals, randomly selected. Since pedestal and signal had the same frequency and phase, the effect of the signal was simply to increase the contrast in the signal interval relative to the no-signal interval. In both paradigms the observer's task was the same, to indicate which observation interval contained the signal grating. Psychometric functions were obtained for both paradigms at three spatial frequencies, 0.5, 2, and 8 c/deg (cycles per degree). A second experiment which employed a paired comparison paradigm will be described below.

Two male observers participated in this study. J.M.F., one of the authors, wore lenses which corrected his acuity to 20/20; G.W. had acuity of 20/20 uncorrected. A session consisted of a block of trials in the detection paradigm and a block of trials in the discrimination paradigm. The order of the two paradigms was alternated from session to session. The three frequencies were presented to J.M.F. in the order 0.5, 2, 8 c/deg and to G.W. in the reverse order.

After the psychometric functions were determined, discrimination over the same range of near threshold contrasts was explored using the method of paired comparisons. Four values of contrast were used, 0 and three other values within the range of the psychometric function. On each trial two different values were randomly selected and randomly assigned to the two observation intervals. The observer indicated which interval contained the grating of higher contrast. Each of the six pairs of gratings was presented approx. 300 times and the proportion of trials in which the higher contrast was correctly indicated was determined. One of the observers, J.M.F., participated in the paired comparison experiment. The same three spatial frequeicies were used in the order: 8, 2, 0.5 c/deg.

RESULTS

The obtained psychometric functions for both detection and discrimination are plotted in Fig. 1.* The detection function describes performance in detecting the presence of a grating; the discrimination function, performance in discriminating which of two contrasts is greater, when the lower contrast on every trial is a contrast approximately in the middle of the detection function. Pedestal contrast is indicated by the solid arrow. Note that the variable plotted on the abscissa is contrast, not contrast difference, so the discrimination function starts at the contrast of the pedestal. The detection functions have a sigmoid form. The discrimination functions rise more abruptly and are considerably steeper. Their lower segments are almost linear. The contrast which yields 75% correct in detection is 3–4 times larger than the difference in contrast that yields 75% correct in discrimination. Open arrows indicate these two contrasts. This confirms the

* A small response bias in favor of the second observation interval was found. J.M.F. selected interval 2 on 57.5% of trials and G.W. on 56% of trials. A bias of this magnitude has a negligible effect on percent correct (Green and Swets, 1966, p. 408).

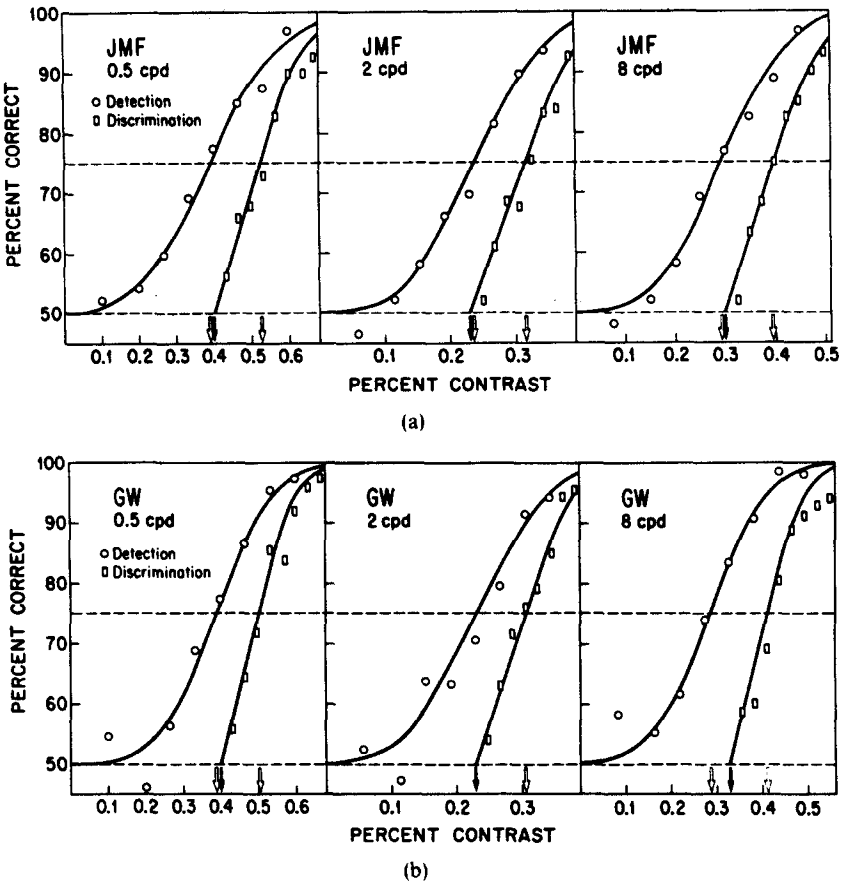

Fig. 1. (a) Forced-choice psychometric functions for contrast detection (○) and near-threshold contrast discrimination (□) for observer J.M.F. at spatial frequencies of 0.5, 2 and 8 c/deg. The open arrows indicate contrasts yielding 75% correct. The solid arrow indicates the contrast of the pedestal present in each observation interval of the discrimination task. See the text for a description of the smooth curves through the data points. (b) Same for observer G.W. The standard deviation of these percentages computed from the percentages in each of the four sessions averaged about 5%.

finding of Nachmias and Sansbury (1974) that near threshold discrimination is better than detection and that the psychometric functions have markedly different forms. Earlier Campbell and Kulikowski (1966) had shown, using the method of adjustment, that discrimination thresholds are lower than detection thresholds. Kulikowski (1976) and Tolhurst and Barfield (1978) obtained the same result using the method of forced-choice.

The smooth curves through the detection data are of the form $p = 100 - 50\exp(-aC^b)$, where p is percent correct and a and b are constants.* They were fitted by the method of least squares omitting the data point at the lowest contrast and sometimes the point at the second to lowest contrast when it deviated enough from the others as to make a close fit impossible. The parameters of the fitted functions and the number of points on which the fit is based are given in Table 1. The parameter b, which reflects the steepness of the function, is close to 3 for all six functions. Goodness of fit would be only slightly reduced if b were taken as 3 for each function. The parameter a reflects relative sensitivity to the different spatial frequencies. It is closely related to the threshold contrast. A high value of a indicates a low threshold. The smooth curves through the discrimination data are not fitted curves. They were theoretical curves whose derivation will be discussed below.

According to Thurstone's model of discrimination (Thurstone, 1927a, 1927b) and commonly used models of sensory detection (Tanner and Swets, 1954; Green and Swets, 1966), the observer's response is determined by the value of a single random variable, D, here called the *decision variable*. This variable has one value for each observation interval and these values are assumed to be independent. A constant contrast, C, is associated with a probability distribution of D whose mean is designated by $F(C)$ and

* This is a variation of the function proposed by Quick (1974) for relating the probability of detection in a yes–no task to contrast. Our dependent variable is not probability of detection on a yes–no task, but rather percent correct in a forced-choice task. It happens, however, that the function fits our forced-choice psychometric functions well. This function has previously been used to fit forced-choice psychometric functions by Legge (1978).

Table 1. Parameters of the function $p = 100 - 50\exp(-aC^b)$ fitted to the detection psychometric function in Fig. 1. p is percent correct; C is percent contrast; a and b are constants. Parameters of the function $-Z(0,C) = (C/C_1)^f$ fitted to the relation between the normal deviate corresponding to the percent correct and the percent contrast in the detection condition. C_1 is the percent contrast needed to yield $Z = 1$

Observer	Frequency	Number of points	a	b	f	C_1
J.M.F.	0.5	7	10.56	2.9	2.48	0.467
	2.0	6	52.60	3.0	2.47	0.285
	8.0	7	25.40	3.0	3.04	0.353
G.W.	0.5	6	18.70	3.5	2.99	0.454
	2.0	6	49.45	2.9	2.11	0.285
	8.0	7	38.90	3.0	2.64	0.344

whose standard deviation is designated by $\sigma(C)$. In a forced-choice task the observer responds by indicating the observation interval for which the value of D is larger. This will be called the *differencing model.* When the two contrasts to be discriminated are designated B and $B + C$, the probability that the observer's response will be correct is the probability that $D(B + C) - D(B) > 0$, where $D(C)$ is the value of the decision variable occurring in the observation interval during which the contrast is C. If D has a Gaussian distribution, this difference also has a Gaussian distribution with mean $F(B + C) - F(B)$ and variance $\sigma^2(B + C) + \sigma^2(B)$. Thus, the percent correct will correspond to the percentage of the area of this difference distribution that lies to the right of 0. The normal deviate of the point at which the difference is 0 (the normal deviate corresponding to the percent correct) is related to $F(C)$ and $\sigma(C)$ as follows:

$$Z(B, B + C) = -[F(B + C) - F(B)]/[\sigma^2(B + C) + \sigma^2(B)]^{1/2}. \quad (1)$$

If the standard deviation is constant over the range of the psychometric function, that is if $\sigma(C) = \sigma$:

$$-Z(B, B + C) = [F(B + C) - F(B)]/\sqrt{2}\,\sigma \quad (2)$$

Later, evidence will be given concerning the constancy of $\sigma(C)$ over the range of these functions. Since percent correct will generally be greater than 50, Z will generally be negative and $-Z$ will be positive. When $\sigma(C)$ is constant, $-Z$ is the difference between the mean values of the decision variable corresponding to the two contrasts B and $B + C$ measured in units corresponding to $\sqrt{2}\,\sigma$. The difference between mean values of the decision variable is often expressed in units corresponding to σ and is then called d'. Thus, when standard deviation is constant:

$$d'(B, B + C) = -\sqrt{2}\, Z(B, B + C). \quad (3)$$

From equation 2, letting $F(0) = 0$, the value of d' for detection, $d'(0, C)$, is:

$$d'(0, C) = d'(C) = -\sqrt{2}\, Z(0, C) = F(C)/\sigma. \quad (4)$$

Both $-Z(B, B + C)$ and $d'(B, B + C)$ are commonly referred to as psychometric functions, even though they are not linearly related to percent correct. To distinguish them from the percent correct function and from each other, we will refer to the first as the Z function and to the second as the d' function. These functions are simply transforms of percent correct as a function of C in a two alternative forced choice task.

Thus, according to this model, if we assume equal variance, $-Z(0, C)$ and $d'(0, C)$ are proportional to the mean, $F(C)$, of the decision variable distribution. We will refer to the function $F(C)$, which is defined by equation 4, as the F function. From equations 2 and 4:

$$-Z(B, B + C) = -Z(B + C) - [-Z(B)] \quad (5)$$

Thus, the model implies that the normal deviate corresponding to percent correct for the discrimination of two contrasts is the difference between the normal deviates for the detection of the same two contrasts. This property has been referred to as Z additivity (Pelli, 1979).

The Z function is shown by the open circles in Fig. 2 for each of the six detection psychometric functions of Fig. 1. The normal deviate can be seen to be a concave upward function of contrast. This relation was fitted with a function of the form:

$$-Z(0, C) = (C/C_1)^f \quad (6)$$

where C_1 is the contrast which yields $Z = 1$ (84% correct) and f is a constant. The same upper 6 or 7 points were used to fit the curves as were used to fit the curves in Fig. 1. Parameters of the fitted curves are given in Table 1. The parameter f averages about 2.6 and shows no clear trend with spatial frequency, although both observers show a minimum at 2 c/deg. If still fewer points are used, agreement among values of f improves. For example, if only six points are used at 8 c/deg, the value of f is 2.49 for both subjects. A value of $f = 2.5$ gives a good fit to all functions except at the lowest contrast values, where the data are more variable.

Nachmias and Sansbury (1974) fitted a function of the same form to data on forced-choice detection of a 9 c/deg grating. Their exponents were 2.2 and 2.9 for their two observers. Using a confidence rating method

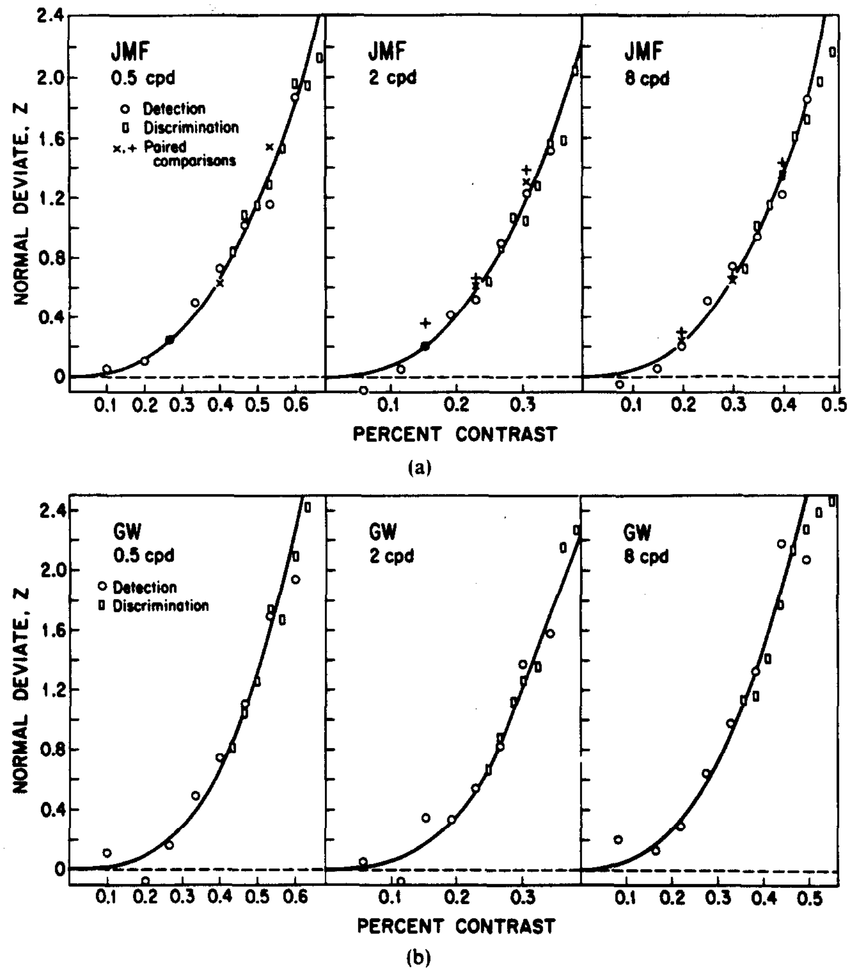

Fig. 2. (a) (○). Normal deviate transform of percent correct (Z function) in detection task as a function of percent contrast. (□). Normal deviate transform of percent correct in discrimination task plus the normal deviate of the pedestal contrast derived from the detection data. (×). Normal deviates derived from the paired comparisons data using the equal variance model. (+). Normal deviates derived from the paired comparisons data using the variable variance model (*F* function). (b) Same for observer G.W. G.W. did not participate in the paired comparisons experiment

to study detectability of a 9 c/deg grating, Stromeyer and Klein (1974) obtained quite comparable results. They assumed that standard deviation increases at 1/4 the rate of the mean and fitted their data with a more complicated function in which the exponent decreases from 4 to 2 as contrast increases. Van Meeteren (1978) also found a positively accelerating relation between d' and grating contrast using a yes–no paradigm. In all of these experiments contrast remained constant during a block of trials. This procedure leaves open the possibility that the nonlinearity is due to a decrease in criterion variance as contrast increases (Wickelgren, 1968; Nachmias and Kocher, 1970). When the contrast is varied randomly from trial to trial, as in the present experiment, there is no way in which criterion variance could vary with contrast. Sachs *et al.* (1971) reported an extensive study using a yes–no procedure. They fitted their data with a Z function that was linear with contrast together with a correction for guessing. Without the correction, the relation between Z and C was positively accelerating in most cases. However, in 17 of 41 data sets no correction for guessing was needed. These particular data sets appear to be at variance with the rest of the data. A number of investigators have obtained psychometric functions consistent with positively accelerating Z functions from data on luminance increment detection using forced-choice, yes–no, and rating scale experiments (Tanner and Swets, 1954; Leshowitz *et al.*, 1968; Nachmias and Kocher, 1970; Cohn *et al.*, 1974). Exponents of fitted power functions range from 1.5–3.3.

If Z additivity (equation 5) holds, then the Z function for discrimination should correspond to the upper portion of the Z function for detection. To test this the value of Z (from the fitted curve) corresponding to the pedestal contrast was added to each normal deviate derived from the discrimination experiment. These values are plotted as rectangles in Fig. 2. They lie very close to the Z function derived from the detection data.

It is now possible to explain how the curves

Table 2. Paired-comparisons experiment. Percent correct for each pair of grating contrasts. Observer J.M.F. $n \approx 300$

Spatial frequency (c/deg)	Contrast pair (Percent contrast)	Percent correct
0.5	0.533, 0.400	82.2
	0.533, 0.267	90.1
	0.533, 0	94.1
	0.400, 0.267	64.6
	0.400,0	76.3
	0.267, 0	60.2
2.0	0.307, 0.230	77.7
	0.307, 0.153	87.3
	0.307, 0	88.4
	0.230, 0.153	64.7
	0.230, 0	76.0
	0.153, 0	59.6
8.0	0.400, 0.300	76.2
	0.400, 0.200	88.4
	0.400, 0	89.8
	0.300, 0.200	62.9
	0.300,0	76.5
	0.200, 0	56.9

through the discrimination functions in Fig. 1 were obtained. The fitted Z function of Fig. 2 was used to predict these functions using equation 5. The value of Z at each of the contrasts (constant pedestal plus increment) used in the discrimination task was determined from this fitted function. The normal deviate of the pedestal was subtracted from this value and the resulting value of $Z(B, B + C)$ transformed to percent correct. These values correspond to the smooth curves through the discrimination data in Fig. 1. The fit is quite good. Thus, the Z function can be used to predict near-threshold discrimination performance from detection performance without any additional parameters. This was first shown by Nachmias and Sansbury (1974) for 9 c/deg gratings. They, however, had only 3 and 5 points on their two discrimination functions and the fit was not as convincing as in the present case. Sansbury (1977) obtained a similar result using a square-wave masker. The fact that the same Z function describes both detection and discrimination data precludes the possibility that there is a large enough change in decision variable variance to account for the nonlinearity of the Z function. A large difference between $\sigma(0)$ and $\sigma(B)$ would cause Z to increase less rapidly with contrast in the discrimination paradigm, since the denominator of equation 1 would be larger.

The paired comparisons experiment provides another test of the form of the Z function, Z additivity, and a more powerful test of the equal variance assumption. Here the data are the six values of percent correct corresponding to the six pairs of contrasts used in the experiment. These values are presented in Table 2. They were used to compute the normal deviates corresponding to the difference between zero and the three non-zero contrasts and to test the equal variance model. The data were analyzed in the manner of Thurstone's Case V. (Thurstone, 1927a; Togerson, 1958). In this analysis percent correct for each pair of stimuli is transformed into the corresponding normal deviate. These normal deviates are taken as estimates of the differences in mean decision variable for each of the pairs of stimuli. The method of least squares is used to find a single value for the difference between each pair of stimuli. The resulting values of Z are plotted as X's in Figure 2. They lie quite close to the Z function derived from detection. A χ^2 test of goodness of fit (Bock and Jones, 1968, p. 135) showed that the equal variance model fits the data almost perfectly at 0.5 c/deg, but that deviations from this model were barely significant at the other two frequencies. At 2 c/deg $\chi^2 = 11.38$, $df = 3$, $P < 0.01$. At 8 c/deg $\chi^2 = 10.27$, $df = 3$, $P < 0.025$. Thurstone's case IV analysis (Torgerson, 1958) was then carried out. This estimates the ratios among the standard deviations of the decision variable at different contrasts as well as the values of the Z function corrected for the change in standard deviation, i.e. $F(C)$. Values of the estimated ratio of standard deviation at each contrast to the SD at zero contrast are given in Table 3. It can be seen that as contrast increases standard deviation at first decreases and then increases again. This is contrary to the usual finding for light detection, which is that standard deviation increases in proportion to the change in the mean with the constant of proportionality being about 0.25 (Nachmias and Kocher, 1970; Swets *et al.*, 1961). Stromeyer and Klein (1975) reported a value of 0.27 in an experiment on grating detection. In all three studies, however, the conclusion is based on the slope of the ROC curve. This parameter is influenced by changes in criterion variance (Wickelgren, 1968), a factor that seems unlikely to be important in the forced choice paradigm. However, the present study is not extensive enough to be decisive with respect to

Table 3. Estimates of the ratio of standard deviation of the decision variable at contrast C to the standard deviation at zero contrast, $\sigma(C)/\sigma(0)$

Frequency (c/deg)	Percent contrast	$\sigma(C)/\sigma(0)$
0.5	0	1.00
	0.27	1.02
	0.40	1.01
	0.53	1.01
2.0	0	1.00
	0.15	0.77
	0.23	0.70
	0.31	0.92
8.0	0	1.00
	0.20	0.79
	0.30	0.85
	0.40	1.18

Table 4. Models of detection and near-threshold discrimination considered in this paper

	Nature of Nonlinearity: Monotonic	Nature of Nonlinearity: Nonmonotonic (Threshold)
Nonlinear Transducer (noise originates centrally)	Monotonic transducer followed by noise (cross-hatched)	Threshold alone (cross-hatched)
		Threshold followed by noise
Uncertainty (noise originates in sensory channels)	Likelihood ratio with signal uncertain (cross-hatched)	Maximum response with signal uncertain (cross-hatched)
	Likelihood ratio with signal and pedestal uncertain	Maximum response with signal and pedestal uncertain

Note. All the models involve a nonlinear transformation of sensory signals followed by the addition of noise. Each model is consistent with a positively accelerating detection Z function. Only those which are not cross-hatched are consistent with detection, discrimination, and noise masking results.

the standard deviation function. What does seem clear is that any change in the standard deviation over the range of the psychometric function is relatively small, too small to account for or even greatly influence the form of the relation between the normal deviate and contrast. This is illustrated by the plus symbols (+) in Fig. 2 which indicate the value of the F function yielded by the case IV analysis, which uses the varying estimates of standard deviation in computing the relation between mean decision variable and contrast. The values are seen to be only slightly different from those computed using the equal standard deviation assumption. In either case the F function has a marked positively accelerating nonlinearity near threshold.

In this paper we are concerned with only a very small part of the range of the F function at the lower end. It is possible to apply the same approach over the entire range of the function. We have done this as part of a more general study of contrast masking (Legge and Foley, 1980). Above threshold the F function becomes decelerating, that is, it shows a compressive type of non-linearity.

The results may be summarized as follows. Z functions were found to be power functions having exponents of about 2.5. Z additivity was obtained. The data were well fitted by a model which assumes a differencing decision rule, Gaussian decision variable distributions of approximately constant variance and a nonlinear relation between mean decision variable and near-threshold contrast.

DISCUSSION

The differencing model with a positively accelerating F function and approximately constant variance accounts for contrast detection and near-threshold discrimination at one level of analysis. It attributes the form of the d' or Z functions for both detection and discrimination to a positively accelerating nonlinearity in the F function. Yet, in the literature on detection, hypotheses and models are found which go further in that they specify processes that underlie the nonlinearity of the Z function. Among these models are uncertainty models, which attribute the nonlinearity to the observer's lack of knowledge of stimulus parameters. These are opposed to simpler ideas that we will refer to as nonlinear transducer models. It is of interest to consider how our results bear on these models.

We will consider several models. None of these are fully specified in the literature for forced-choice detection and discrimination. They are extensions of ideas found in the literature. Table 4 summarizes the models that we will consider.

These models attempt to explain why $F(C)$ is positively accelerating and $\sigma(C)$ is approximately constant. Since the stimulus field is large and its average luminance remains constant throughout, it seems reasonable to assume that the variance due to quantum absorption is approximately Gaussian and remains constant for low contrast grating stimuli. The problem is to discover what process introduces a nonlinearity in $F(C)$, while $\sigma(C)$ remains constant. Any nonlinear transformation of the decision variable will affect both $F(C)$ and $\sigma(C)$ and the form of the decision variable distribution. However, any monotonic transformation will have no effect on Z, since performance depends only on the sign of the difference of the two values of the decision variable obtained on each trial, and a monotonic transform will leave this sign unchanged. A consequence of this is that, although we have shown that our data are consistent with a decision variable that has a Gaussian distribution, any monotonic transformation of this variable produces another model that describes the data equally well.

Nonlinear transducer models

A monotonic nonlinear transducer* which is followed by the addition of Gaussian noise provides a simple model which adequately describes our data. The amplitude of the post-transducer noise must be large relative to the pretransducer noise to make the decision variable variance approximately constant.

A nonmonotonic transducer that produces the same output for all inputs up to some threshold value and a linear input-output relation above this value (threshold transducer) affects both $F(C)$ and $\sigma(C)$, but it affects them differentially so that Z is a positively accelerating function of contrast. If the threshold is set so that it is rarely exceeded at 0 contrast, then in the detection case this model reduces to the high threshold or double-detection model (Blackwell, 1963; Treisman and Leshowitz, 1969). In the high-threshold model the decision rule is to select the interval in which the decision variable exceeds the threshold and, if neither exceeds it, to respond randomly with the two responses having equal probability. In the detection case, since the threshold is rarely exceeded by noise alone, this rule is equivalent to a differencing rule with the provision that should the difference be 0, the response will be random. The presence of the threshold reduces the percent correct below what it would be in the absence of the threshold. The reduction is greatest at low contrasts where the observer frequently responds randomly. The result is a positively accelerating Z function. The goodness of fit of this model to our detection results was examined in the following way. According to this model the percent correct is related to the percentage of trials on which the threshold is exceeded (percent detected p_d as follows (Green and Swets, 1966, p. 129):

$$p = p_d + 0.5(100 - p_d) = 0.5(100 + p_d), \quad (7)$$

so that the percentage of trials on which the signal is detected is:

$$p_d = 2p - 100. \quad (8)$$

We call the sensory signal prior to the threshold the input. Input is a Gaussian random variable with constant variance. Mean input $r(C)$ is proportional to contrast:

$$r(C) = gC. \quad (9)$$

The percent detected, p_d, corresponds to the percentage of the area of the decision variable distribution that exceeds the threshold. Consequently, the normal deviate corresponding to the percent detected, Z_d, is related to the mean of the input distribution $r(C)$ measured in standard deviation units as follows:

$$-Z_d = r(C) - I_t = gC - I_t, \quad (10)$$

where I_t is the magnitude of the input at threshold measured in standard deviation units relative to the mean of the 0 contrast distribution. We fitted the relation between Z_d and C with a straight line. The fit was quite good. However, this model predicts that the ROC curve for contrast detection will be a straight line, contrary to the results of Stromeyer and Klein (1975).

The inadequacy of the threshold transducer model to our own data becomes apparent when we consider the discrimination results. Here the pedestal will be detected half the time, since the pedestal is chosen so that p in equation 8 is about 75. The pedestal plus signal will be detected half the time or more, depending on the signal contrast. Thus, a random response will occur on at most about 25% of trials and this percentage will decrease as signal contrast increases. As a consequence, the nonlinearity of the Z function will be considerably lessened. The predictions of the model cannot be readily derived analytically, but we have examined them by means of a computer simulation. The model produces a nearly linear Z function for discrimination which is shallower than the upper region of the detection function. Thus, the model predicts a failure of Z additivity. It also predicts that $\sigma(C)$ increases with contrast. Since these predictions do not correspond with our results, this simple threshold transducer model is rejected.

A threshold transducer which is followed by the addition of noise is consistent with our results. The noise amplitude must be large relative to the pretransducer noise so that decision variable variance is approximately constant. Figure 3 depicts a model which incorporates a nonmonotonic transducer. The input first passes through a linear spatial frequency filter whose mean output is proportional to contrast for a constant input waveform. Although the present result does not implicate such a filter, there are numerous studies that do. See, for example, Braddick *et al.* (1978). A model must incorporate such a filter if it is to describe performance in which there are masking stimuli of different frequencies from the signal (Legge and Foley, 1980). The output of the linear filter is the input to the nonmonotonic transducer. This input is assumed to be noisy due to the quantal fluctuation of light emission. As a consequence, the mean output of the transducer is a positively accelerating function of contrast. Noise is then added to make the variance of the decision variable approximately constant. It is assumed that neither the transducer nor the noise amplitude are influenced by the momentary stimulus or the observer's cognitive state.

Lasley and Cohn (1981) have shown that noise masking experiments provide a basis for testing the monotonic transducer followed by noise model. The

* In this context "nonlinear transducer" refers to a nonlinear transform imposed on the input. Although this term is frequently used in this way in discussions of sensory detection, "nonlinear transform" would appear to be preferable, since there is no implication that the receptor cells (transducers) are the nonlinear elements. In general, the transducer function is not identical to the F function, which relates the mean decision variable to contrast, although in the monotonic nonlinear transducer model the two functions are the same.

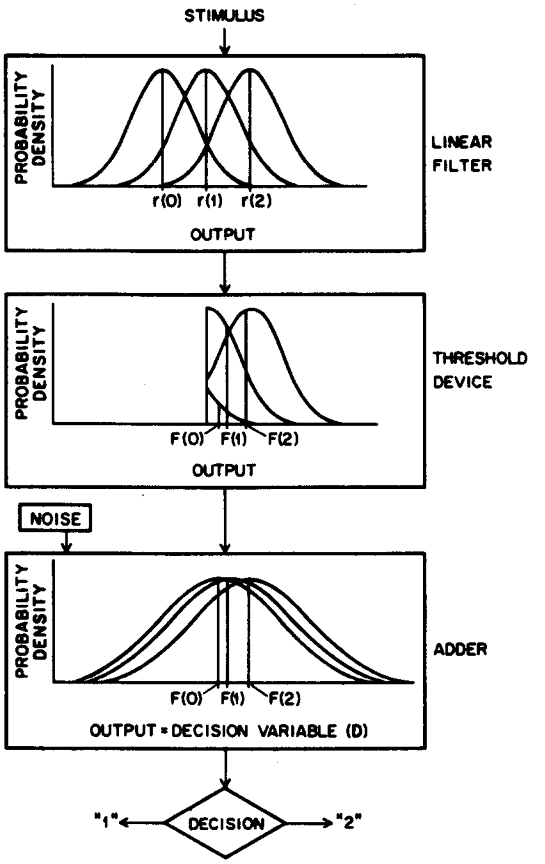

Figure 3. Block diagram of nonlinear transducer model in which the non-linearity is produced by a threshold device followed by the addition of Gaussian noise. The three distributions in each panel correspond to the distributions of central events associated with three equally spaced values of contrast 0, C_1, and C_2.

properties of this model depend on the fact that the pre-transducer noise amplitude is small relative to the noise added after the transducer. If large amplitude noise is added to the stimulus, the noise added after the transducer will have little consequence and the model will perform like a monotonic transducer without added noise. The monotonic model will yield a linear Z function. The threshold transducer model, on the other hand, will continue to yield a positively accelerating Z function. The nonlinearity, however, will decrease as noise amplitude increases. Pelli (1979) studied the detection of gratings in the presence of continuous dynamic visual noise. He showed that Z function nonlinearity does not disappear for a noise amplitude sufficient to raise the contrast threshold by a factor of 10. He describes the shape of the psychometric functions as being unchanged by the noise. This result rejects the monotonic transducer model. The implications of this experiment for the non-monotonic transducer model require a more careful examination. Pelli's noise was continuous and, consequently, it would seem likely to have produced adaptation as well as masking. Up to this point we have assumed that the threshold remains fixed. However, a model in which the threshold shifts with the state of adaptation is not altogether implausible. Even if the Z function remains constant in form as noise amplitude increases, a model in which the threshold varies with the state of adaptation is not excluded.

Uncertainty models

Uncertainty models have been presented as a possible way to reconcile linear cellular responses in the peripheral visual system with the nonlinearity of the d' function (Nachmias, 1972). However, uncertainty models are like nonlinear transducer models in that both incorporate a nonlinear operation preceding the decision. Neither type of model attempts to place this nonlinearity anatomically and both are compatible with a central locus. Commonly, the non-linearity is described as preceding the decision making process in the non-linear transducer models and as part of the decision making process in the uncertainty models. This distinction, however, has no empirical consequences in the kinds of experiments we are considering. In each case the nonlinearity is followed by the addition of a noisy input.* Thus, the uncertainty models have the same general form as the nonlinear transducer models. The critical difference between the uncertainty models and the nonlinear transducer models is that in the uncertainty models the additive noise arises from sensory events that are insensitive to the signal. This means that the additive noise is subject to influence by a stimulus, such as a masker, which accompanies the signal. The contribution of these events is attributed to the observer's uncertainty regarding stimulus parameters. The term "uncertainty" suggests that the input from these insensitive events is subject to influence by cognitive factors, but the operations that will cause subjective uncertainty to vary are not specified. The uncertainty models also differ from the nonlinear transducer models in the nature of the nonlinear transform.

Two kinds of uncertainty models have been proposed. These are *likelihood ratio models* in which the decision variable is a likelihood ratio (Peterson *et al.*, 1954; Tanner, 1961; Cohn *et al.*, 1974; Green and Birdsall, 1978) and *maximum response models* in which the observer monitors M channels, one of which is sensitive to the signal, and in which the decision variable is the maximum response obtained from all channels in each observation interval (Wainstein and Zubakov, 1962; Nachmias and Kocher, 1970). If the inputs to the decision process are assumed to have Gaussian distributions of equal variance, the two

* This common form of the models is explicit for all the models except the maximum response models. The maximum response models also have this form in the following sense. A threshold nonlinearity in each channel, with the threshold set at the maximum response occurring in the first observation interval, followed by summing the outputs across channels, will yield the same decisions as a maximum response decision rule.

Table 5. Signal uncertainty model. Detection: values of parameters M and g. Discrimination: Values of the parameters C_1, f, M and g. Values determined by fitting functions using the methods of least squares

Observer	Spatial frequency (c/deg)	Detection M	g_{det}	Discrimination C_1	f	M	g_{disc}	g_{disc}/g_{det}
J.M.F.	0.5	296	7.55	0.181	1.05	1.3	8.23	1.09
	2.0	285	12.35	0.119	1.43	7.2	20.39	1.65
	8.0	2032	10.84	0.137	1.56	11.8	19.22	1.77
G.W.	0.5	1737	8.36	0.151	1.25	3.5	13.65	1.63
	2.0	64	11.63	0.101	1.30	4.3	16.64	1.43
	8.0	534	10.62	0.128	1.12	1.9	12.56	1.19

kinds of uncertainty models produce very similar predictions (Nolte and Jaarsma, 1967).

Likelihood ratio uncertainty models are derived from models of ideal detectors. An ideal detector is one that yields the best possible performance in a detection task, limited only by the variability of the physical stimuli. Likelihood ratio models of visual detection are generally degraded versions of ideal detectors in that they are characterized by receptor quantum efficiencies less than 1 (not every photon produces a visual effect), and intrinsic noise (Nachmias, 1972). However, they retain the optimal decision rule, which for two-alternative forced-choice is a differencing decision rule in which the decision variable is a likelihood ratio, or what is equivalent, a monotonic transform of the likelihood ratio. A likelihood ratio is the ratio of the probability of some event, given that the signal has been presented, to the probability of the same event, given noise alone.* Such a detector must identify events precisely and then determine their likelihood ratios by referring to stored probability distributions. It is assumed that the events I_i on which the decision is based may be represented by Gaussian random variables of equal variance, with mean r_i being a linear function of contrast. The detection process may be described as follows. First, a likelihood ratio is computed for the event associated with each of the M possible signals. This ratio is computed using the distribution that I_i would have if signal i were present. This likelihood ratio is related to I_i and d' (the difference between the means of the I_i distributions with and without the signal) as follows (Green and Swets, 1966, p. 60):

$$l(I_i) = \exp(d'I_i). \tag{11}$$

This is a monotonic positively accelerating transformation of I_i. The M likelihood ratios are then added together and divided by M to give the overall likelihood ratio $l(I)$:

$$l(I) = (1/M)\sum_i^M \exp(d'I_i), \tag{12}$$

where $I = I_1, I_2, \ldots I_M$. This overall likelihood ratio is the decision variable in the likelihood ratio model. It is a positively accelerating function of contrast and its variance also increases with contrast due to the exponential transform (Peterson *et al.*, p. 207). Since both mean and variance increase and the distributions are not Gaussian, the implications of this model for the form of the Z function are not obvious. It can be shown that for $M = 1$, the Z function is linear. In this case the model reduces to a monotonic transformation of the input which, as we have pointed out, has no effect on performance. For $M \geq 2$, the Z function is positively accelerating and becomes increasingly so as M increases. Analytically, the dependence of Z on M has been described only by an approximation (Peterson *et al.*, 1954, p. 207). However, exact values have been computed for values of M from 2 to 100 (Nolte and Jaarsma, 1967). We extrapolated from their values to fit this model to our detection data. The fit is about as good as that of the differencing model (Fig. 2). Values of the parameters M (the number of potential, orthogonal, equally detectable, equally likely signals) and g the constant of proportionality between $r(C)$ and C (equation 9) are given in Table 5.

The second kind of uncertainty model is the maximum response model. This is an extension of the model of Wainstein and Zubakov (1962) and Nachmias and Kocher (1970) to forced-choice detection and discrimination. This model was developed for yes–no detection tasks. According to this model, the observer monitors the output of M channels, only one of which is affected by the signal. A criterion is set so that, if the output of every channel is less than the criterion, the decision is that no signal is present, but if the output in at least one channel exceeds the criterion, the decision is that a signal is present. This model is extended to forced-choice by assuming that in each observation interval the maximum response is determined. This maximum response is the decision variable. The interval is selected for which this maximum response is largest. If the distributions of channel responses I_i are Gaussian and of equal variance, this model predicts very nearly the same performance as the likelihood ratio uncertainty model (Nolte and Jaarsma, 1967). Therefore, it gives an equally good fit

* The phase "some event" is left deliberately vague in these models. In principle, the event may be distributed over space and time. In certain ideal detector models, the event corresponds to the value of the cross-correlation between the expected signal and the obtained waveform.

to the contrast detection results with the same value of the parameter M.

The literature contains very little discussion of the application of uncertainty models to pedestal discrimination taks. Cohn *et al.* (1974), working on luminance detection, found that although their detection d' function was nonlinear, discrimination from a pedestal near the detection threshold produced a linear function. They interpreted this result, following a suggestion made by Tanner (1961), as indicating that the nonlinearity was due to uncertainty and that the pedestal eliminated the uncertainty. In the present study discrimination d' functions are not linear, but are more nearly linear than the detection functions. By a generalization of Cohn *et al.*'s hypothesis, this might be attributed to a reduction of uncertainty. The uncertainty reduction hypothesis does not constitute a complete model of discrimination performance. It leaves unspecified whether there is uncertainty for the signal only or uncertainty for both pedestal and signal. The hypothesis suggests that the system has information about the pedestal. If the pedestal were as uncertain as the signal alone, how could it reduce the uncertainty of the signal? Furthermore, it turns out that if pedestal and signal are equally uncertain, our results can be explained without resort to uncertainty reduction. We have examined both *signal-only uncertain* models and *signal-and-pedestal uncertain* models in relation to our results.

In a likelihood ratio model performance depends on how the likelihood ratios are computed. This in turn depends on what the system takes the signal and no signal distributions to be. As has been noted, in the detection case the likelihood ratios are computed as if each of the M signals were present. In the pedestal case, the signal and no-signal distributions associated with the sensitive event have a constant added to each value. In the signal-only uncertain model, the no-signal distribution produced by the pedestal is used to compute the likelihood ratio for the sensitive event only and not for the insensitive events. This produces likelihood ratio distributions that are the same with the pedestal as without. Consequently, the Z function will have the same form with and without the pedestal unless uncertainty changes. Thus, this model can account for the obtained change in the form of the Z function when a pedestal is present only with the additional assumption that the pedestal produces a reduction in uncertainty. We compared our data to the expectations of this model by using the form of the discrimination Z function to determine the parameters M and g in just the same way as we had done for the detection data. Discrimination should yield a lower value of M, but the same value of g as the detection data. Values of these parameters are given in Table 5. Contrary to the expectations of this model, values of g are consistently higher for discrimination than for detection. Thus, this likelihood ratio uncertainty model with uncertainty for the signal alone is rejected. The corresponding maximum response model is one in which the response of the sensitive channel is specified relative to the pedestal alone distribution, so that a shift in this distribution produced by addition of a pedestal has no effect on performance. To specify the response in this way, the system would have to have information concerning the pedestal alone distribution. Since the pedestal has no effect on performance, the predicted form of the Z function is the same as for detection and essentially the same as that inferred from the likelihood ratio model. Hence, this model is rejected by the result described in Table 5.

The other way to generalize the signal uncertainty models to the discrimination paradigm is to assume that the observer is uncertain of the pedestal as well as the signal (*signal-and-pedestal uncertain models*). In the likelihood ratio model, likelihood ratios are computed as if the pedestal influenced each of the M events, i.e. as if the distribution of each I_i were increased by a constant. Since there is actually only one event which is influenced by the pedestal and signal, the likelihood ratios associated with the other events will become increasingly smaller as pedestal contrast increases, and the one sensitive event will increasingly determine the overall likelihood ratio. Thus, even though uncertainty remains constant, the effect of uncertainty decreases as the pedestal contrast increases. Here the prediction of the pedestal effect is inherent in the model.

The corresponding maximum response model assumes that no account is taken of the pedestal in determining the maximum response. As pedestal contrast increases, the maximum response will tend to occur more and more frequently in the one sensitive channel. Thus, the effect of the uncertainty will decrease even though all M channels continue to be monitored. Pelli (1980) has computed the predictions of this model for our observer J.M.F. and has shown that it gives a reasonably good fit. He used the same values of M as are given in Table 5. This model does not predict Z additivity exactly, but it does predict approximate Z additivity over the range of our data.

Since all of the models considered have a common form, a nonlinearity followed by noise, and some of these are consistent with our results, we conclude that the results can be explained by such a process. It is possible that the system is more complicated than any of these models, perhaps by combining elements from several of them. The principle difference among the models is whether the noise arises centrally (nonlinear transducer models) or peripherally in sensory channels that are insensitive to the signal (uncertainty models). Recent experiments indicating that this noise is not "swamped" by masking noise favor the latter alternative.

The possibility that our results might be accounted for by stimulus uncertainty raises several questions. (1) Since it is demonstrable that human observers can identify stimuli at threshold at least on some dimensions (e.g. Thomas, 1978; Burr, 1980), why is this information not used in detection tasks? (2) What are the dimensions of uncertainty? The signal and pedestal uncertainty models imply that even a low contrast pedestal which *differs* from the signal along a dimension of uncertainty will greatly reduce performance. Legge and Foley (1980) have shown that pedestals of different spatial frequency from the signal do not reduce performance until their contrast is well above threshold. This suggests that uncertainty along the spatial frequency dimension is not responsible for our results. Onset and offset uncertainty would be expected to have larger effects for short duration stimuli, yet, in general, psychometric functions have the same form over a wide range of durations (Green and Luce, 1975), although this has not yet been shown for grating detection. (3) What operations, if any, will reduce uncertainty? We expected that, if uncertainty were a factor in these experiments, the suprathreshold glimpse of the stimulus which was presented just prior to each trial would reduce it. Yet our Z functions are just as nonlinear on the average as those of Nachmias and Sansbury (1974), who did not use this technique. (4) If thresholds depend on uncertainty as well as the sensitivity of the detecting channel and uncertainty varies with the stimulus and the perceiver's cognitive state, can thresholds provide an accurate indication of the sensitivity of the detecting channel? We conclude that, although the uncertainty models appear to be the most viable at the present time, they pose a number of problems that can only be answered by further experimentation.

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