Tolerance to visual defocus

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Low-resolution optical systems are more tolerant to defocus than are high-resolution systems. We wished to determine whether this principle applies to human vision. We used psychophysical methods to measure the effects of defocus in normal eyes under low-resolution conditions. Modulation transfer of sine-wave gratings was measured as a function of dioptric defocus at low and medium spatial frequencies. We defined the depth of focus at a given spatial frequency to be the dioptric range for which the modulation transfer exceeds 50% of its peak value. For dilated pupils, depth of focus increased from about 2.5 diopters (D) at 3.5 cycles/deg to about 17 D at 0.25 cycles/deg. From our results we predicted that tasks requiring only low spatial frequencies will be more tolerant to defocus than tasks requiring higher spatial frequencies. This prediction was confirmed in a letter-recognition experiment. The increasing tolerance to defocus at low spatial frequencies also implies that individuals with low acuity will be more tolerant to defocus than people with normal vision. We confirmed this prediction by measuring tolerance to defocus in 30 low-vision eyes.

INTRODUCTION

Low-resolution optical systems are more tolerant to defocus than are high-resolution systems.[1] If this principle applies to vision, we would expect people with low acuity to be more tolerant to defocus than would people with normal vision. We investigated this question by measuring tolerance to defocus for subjects with normal vision under low-resolution conditions, that is, at low and medium spatial frequencies. We used our results to predict the effects of defocus on the acuity of subjects with both normal and low vision. Our findings are pertinent to the prescription of eyeglasses and reading aids for people with low vision.

Several methods have been used to evaluate the effects of lens defocus on human vision. These include (1) detection of blur,[2] (2) reduction of contrast sensitivity,[3–5] (3) reduction of acuity,[6–9] (4) reduction of vernier acuity,[10] (5) detection of the direction of speckle motion in a laser optometer,[11] and (6) photodetector measurements of line-spread functions.[12]

In one of the first experimental studies of visual defocus, Campbell[2] used targets consisting of black disks 10 arcmin in diameter mounted on glass plates in front of a uniform white screen. While a subject fixated on a disk at a distance of 50 cm, a second disk, immediately to the left or right, was moved forward or backward along an optical rail until it appeared blurred. The near and far positions of just-noticeable blur determined the depth of field. The dioptric distance of these points from the point of fixation constitutes a measure of the depth of focus. Campbell examined the effects of luminance, contrast, and pupil size and color on the depth of focus. For a screen luminance of 1000 mL and a 3-mm-diameter pupil, the median depth of field for seven subjects was ±0.43 diopters (D). Foreshadowing later findings, Campbell commented that "it is probable that objects of different size and shape would give different values for the depth of field."

This prediction was borne out in studies using a different method by Campbell and Green[3] and Green and Campbell.[4] They were concerned with the effects of defocus on the modulation transfer function (MTF) of the eye. The retinal image of a sine-wave target closely approximates a sinusoidal intensity distribution[13]; the only effect of defocus is to reduce image contrast (and possibly phase, as discussed below). Therefore any reduction in contrast sensitivity associated with defocus can be interpreted as a measure of the preretinal transfer properties of the eye's optics. Several authors have presented theoretical MTF's for aberration-free, diffraction-limited, defocused eyes.[3,5,12,14,15] Their calculations show that defocus acts to attenuate the contrast more at medium and high spatial frequencies than at low ones. Green and Campbell measured the effects of lens defocus on the contrast sensitivities of subjects for sine-wave gratings of different spatial frequencies. In keeping with the theoretical predictions, it was found that a given amount of defocus reduced contrast transfer more at high spatial frequencies than at low ones. Charman[5] conducted more detailed measurements of the same kind and obtained very similar results.

We can use the data of Green and Campbell[4] to estimate tolerance to defocus. We take as an index the defocus required to reduce contrast transfer by a factor of 2. We refer to this as the half-amplitude defocus. The data of Green and Campbell show that for a subject with a 7-mm-diameter pupil, the half-amplitude defocus at 1.5 cycles per degree (c/deg) is ±1.5 D, and that at 9 c/deg is only ±0.7 D. This difference shows that tolerance to defocus is indeed greater at low spatial frequencies. Green and Campbell did not explicitly address the issue of tolerance to defocus. Most of their data were collected at high spatial frequencies. One goal of our study was to extend the defocus work of Green and Campbell[4] and Charman[5] to low spatial frequencies, interpreting the findings in terms of tolerance to defocus.

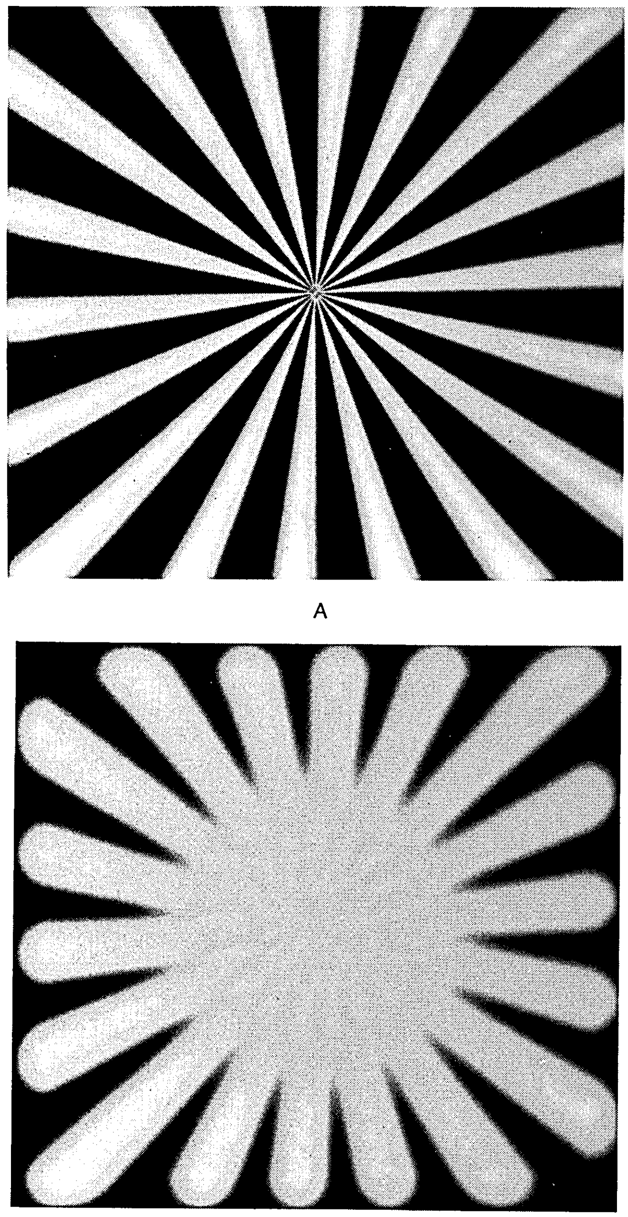
A second goal of our research was to use our modulation-transfer measurements to predict the effects of defocus on letter recognition. Information for recognizing large letters is carried by low spatial frequencies, so we would expect the recognition of larger letters to be more tolerant to defocus. We used data on the spatial-frequency bandwidth requirements of reading and letter recognition[16,17] to predict the tolerances of observers to defocus for letters of different sizes. We compared our predictions with data collected from four normal observers. A forerunner to our findings was supplied by Ogle and Schwartz.[8] They had subjects perform a forced-choice task that involved resolving the squares of a checkerboard pattern. As the check size increased, subjects could maintain a criterion level of performance with more and more defocus. Ogle and Schwartz estimated that there was an additional 0.3 D of tolerance for every 0.25-arcmin increase in check size.

Another prediction follows from the increased tolerance to defocus at low spatial frequencies. Observers who are able to detect only low spatial frequencies, i.e., observers with low acuity, should be more tolerant to defocus. Studies of amblyopes[18] and 6-week-old human infants[19] confirm this prediction. Most people with low vision have substantially reduced acuity. It would be clinically useful if their acuities were predictive of their tolerance to defocus. We measured acuity and tolerance to defocus in 30 low-vision eyes.

Two additional issues will be addressed in this paper. First, we will consider several factors that may contribute to greater tolerance to defocus than would be expected from an ideal, diffraction-limited optical system. We conducted ancillary experiments in which we compared tolerance to defocus in monochromatic and polychromatic light and for vertical and horizontal gratings.

We also considered the possible effects of spurious resolution. Spurious resolution occurs when the modulation transfer function passes through zero and becomes negative.[20] The optical consequence is a phase reversal in the image of a sine-wave grating. Spurious resolution can be observed in an optical system when it is severly defocused, as illustrated in Fig. 1. Figure 1A is a focused television image of a sunburst pattern in which luminance varies sinusoidally around the circle. In Fig. 1B, the camera was defocused. Concentric gray rings mark spatial frequencies of zero contrast in the MTF and separate regions of opposite contrast polarity in the image. Location of the zeros in the transfer function can be predicted from geometrical or wave optics.[20] Westheimer[14] was the first to predict that spurious resolution might be observable in human vision. Although there have been informal demonstrations confirming this,[21] we are unaware of any systematic attempt to measure the location of zeros in the human MTF under conditions of defocus. We conducted a brief experiment with this purpose. We wanted to know what effect spurious resolution might have on our measurements of tolerance to defocus.

Finally, we wished to determine how depth of focus mea-



A

B

Fig. 1. Demonstration of spurious resolution. A, A sinusoidal sunburst pattern has been photographed from a TV screen with a camera in focus. B, The same pattern has been photographed with the camera defocused. The regions of contrast reversal demonstrate spurious resolution. The transitions between these regions demonstrate zeros in the MTF of the defocused optical system. To observe spurious resolution in your own eye, hold Fig. 1A at arm's length and observe it while focusing much nearer.

sured with our MTF method relates to the findings of Campbell,[2] who used just-noticeable blur. We found that data from the literature on contrast discrimination provide a plausible link between the two types of measurement.

METHOD

Effects of Defocus on Modulation Transfer: Threshold Method

Apparatus

Sine-wave gratings were presented on a Joyce Electronics cathode-ray tube (CRT) display with a desaturated green P31 phosphor. Sinusoidal waveforms were synthesized digitally by a Nascom microcomputer. After 10-bit digital-to-analog (D/A) conversion, signals were applied to the Z-axis input of the CRT display. Contrast was controlled by a 12-bit multiplying input to the D/A converter. The screen had a mean luminance of 300 cd/m^2. It was masked down to a circular aperture subtending 3 deg at 300 cm (for experiments at 1–4 c/deg) and 12 deg at 75 cm (for experiments at 0.25 and 0.5 c/deg). In some preliminary experiments, data were also collected at 114 and 57 cm. Viewing distance had no systematic effect on results. Gratings were oriented vertically, except in one experiment in which the CRT was rotated through 90 deg to produce horizontal gratings.

The subject's head was immobilized on a bite bar. Defocus was produced by a trial lens (diameter, 35 mm) mounted approximately 35 mm in front of the cornea. The exact distance was measured in each session. The effect of eye–lens distance on light vergence at the eye was taken into account in computing defocus. We also compensated for the magnifying effect of the defocusing lens.[22] In one experiment, an interference filter (λ_{max} = 577 nm) was placed between the defocusing lens and the eye.

In some experiments, an artificial pupil was mounted close to the eye. Its horizontal and vertical positions were controlled by two micrometer screws. The subject positioned the pupil to achieve maximum contrast for gratings on the screen and for the edge of the circular aperture. It has been shown that if an artificial pupil is decentered perpendicularly to the lines of a grating, both acuity and contrast sensitivity are reduced.[23,24] The effect is probably due to coma or other higher-order optical aberrations for off-axis entry of light rays. However, the effect is small for spatial frequencies of 4 c/deg or less.[24]

In every case the subjects's distance correction was placed in the spectacle plane. In the figures, defocus is taken relative to these corrections.

Procedure

A single session was devoted to contrast-threshold measurements at a single spatial frequency. Measurements were made across a range of positive and negative lens power. The method of adjustment was used. The subject could raise or lower the contrast of a steadily presented grating by pressing one of two buttons. Two to four independent threshold settings were made for each defocusing lens. Standard errors were small, typically 10%, and rarely exceeded 20%.

Subjects

Two of the authors served as subjects (KTM and GW). Both had normal acuity and color vision. For other details, see Table 1. Before each session, accommodation was paralyzed, and the pupil in the right eye was dilated with one drop of 1% cyclopentolate hydrochloride. The left eye was occluded throughout the experiment.

Effects of Defocus on Modulation Transfer: Matching Method

As a check on the basic assumptions and the robustness of the threshold method, we used a matching method to evaluate the effects of defocus on modulation transfer. Figure 2 shows a schematic diagram of the apparatus. Upper and lower hemifields were produced on separate Joyce Electronics CRT displays with white P4 phosphors. The two hemifields were viewed along separate 300-cm-long optical paths by the subject's right eye. The standard display contained a grating with 10% contrast. The contrast of the grating on the variable display could be adjusted by the subject. A defocusing lens was placed in the optical path of the varible display 10 cm from the eye. The subject saw two semicircular fields, one above the other, the total subtending 3 deg. Defocus acted to attenuate the contrast of the variable display. The subject's task was to adjust the contrast of the variable display to match the contrast of the standard. When a match was made, the ratio of variable to standard contrast was assumed to represent the factor by which modulation transfer was reduced by defocus. For example, if the variable contrast was twice the standard contrast when the observer completed a match, modulation transfer was assumed to be attenutated by a factor of 2 because of defocus. Matches were made across a range of positive and negative defocus. Means were based on four or five separate match-

Table 1. Subject Data (Right Eye)

Subject	Age (years)	Distance Correction (D)	Dilated Pupil Diameter (mm)
GW	42	3.25	6.5
KTM	27	2.0	8.0
NS	33	1.5	7.5
JB	21	—[a]	8.0

[a] Emmetrope.

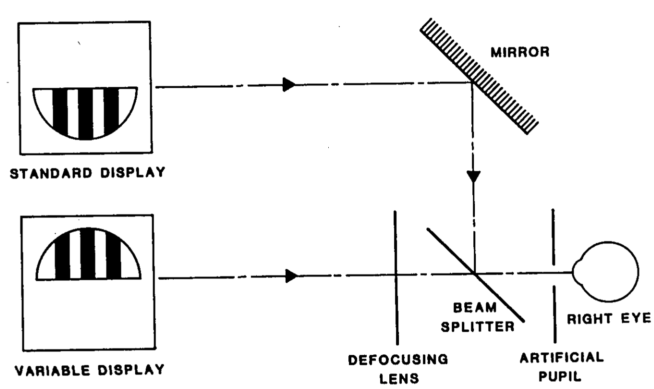


Fig. 2. Schematic diagram of the apparatus used in the matching experiments.

es. The same two subjects participated in the matching experiment.

Measurement of Spurious Resolution

A slide projector was used to display a sunburst pattern like Fig. 1A except that its contours were sharp rather than sinusoidal. Observer KTM viewed the pattern from 6 m through defocusing lenses placed in the spectacle plane. Her pupil was dilated to 8 mm, and accommodation was paralyzed. For some observations, a 2-mm artificial pupil was used. An assistant advanced the tip of a pointer toward the center of the sunburst display until KTM called out that it had reached the location of the first concentric "gray-out" ring in the pattern. The spatial frequency of the periodic light-and-dark pattern, measured along a concentric circular arc, was taken as an estimate of the location of the first zero in KTM's defocused MTF. Measurements of this sort were made for several positive and negative lens powers.

Ideal-Lens Calculations

In the text and figures, modulation transfer of an ideal lens refers to a diffraction-limited lens of the indicated pupil size. Unless otherwise specified, a wavelength of 550 nm is assumed.

Hopkins[1] presented equations from wave optics for the MTF of a defocused lens with a circular pupil. He characterized the amount of defocus by a parameter W_{20} that expresses the optical path difference between a wave front converging on the axial in-focus point and a wave front converging on the axial out-of-focus point. Van Meeteren[25] called W_{20} C_f and expressed it as

$$W_{20} = (n'/2)(az/f^2),$$

where n' is the index of refraction in the image space, a is the pupil radius, f is the focal length in the image space, and z is the defocusing distance in the image space, i.e., the axial distance between in-focus and out-of-focus image planes. This equation can be rewritten in terms of variables in object space as follows:

$$W_{20} = (a^2/2)[DD_0/(D + D_0)],$$

where a is pupil radius, D is defocus in diopters, and D_0 is the dioptric power of the refracting surface. In the formula, a positive value of D corresponds to a positive defocus and an image plane lying a positive distance z in front of the in-focus image plane. We take the total refracting power of the eye to be 66.6 D.[26]

Hopkins's equations cannot be solved analytically, but numerical values have been tabulated by Levi and Austing.[27] They express W_{20} in units of the Rayleigh criterion and call it Δ:

$$\Delta = W_{20}/(\lambda/4).$$

Our calculations of the modulation transfer of ideal lenses were based on the tables of Levi and Austing or, where appropriate, on approximations from geometrical optics.[20]

Effects of Defocus on Letter Acuity: Normal Vision

Letter acuity was measured with the University of Waterloo log MAR (minimum-angle-of-resolution) chart[28] at a viewing distance of 4 m. Letter sizes on this chart are arranged in steps of 0.1 log unit. The chart was illuminated to 270 cd/m^2. Four subjects with normal vision (Table 1) participated. Viewing was monocular (right eye). The pupil was dilated, and accommodation was paralyzed. Subjects wore their normal distance prescription, to which was added a range of positive and negative defocusing lenses, from ±1 to ±16 D. (For some of the stronger lenses, viewing distances of 1 or 2 m were used.)

Tolerance to Defocus in Low-Vision Eyes

Tolerance to visual defocus was studied in 30 low-vision eyes. Subjects selected were patients attending the low vision clinic of the University of Waterloo. All had ocular media or retinal disorders, and none had any neurological pathology. Objective and subjective refractions were performed by using standard clinical techniques. In the event that neither retinoscopy nor a subjective refraction was possible, the method of telescopic refraction was employed.[29] Based upon the findings, a distance prescription was then provided. Best-corrected-distance visual acuity was measured on the University of Waterloo log MAR chart (Version 1). The eye was then defocused with positive lenses until acuity dropped by one line on the chart, representing an increase in recognizable letter subtense by 0.1 log unit. In all cases, natural pupils were used. No patients were included who were under medication to dilate or constrict the pupil.

RESULTS AND DISCUSSION

Effects of Defocus on Modulation Transfer

Figure 3 shows data from several threshold experiments for subject KTM. Figures 3A, 3B, 3C, 3D, and 3E show results for an 8-mm dilated pupil and spatial frequencies of 0.25, 0.5, 1, 2, and 3.5 c/deg, respectively. Figure 3F shows results for a 1-mm-diameter artificial pupil and 3.5 c/deg. Figure 4 shows corresponding data for subject GW. The diameter of his dilated pupil varied from 6 to 6.5 mm, smaller than KTM's. Vertical axes show normalized modulation transfer, which is proportional to contrast sensitivity. The peak value is 1.0. Our technique does not provide values of absolute modulation transfer. The horizontal axis is defocus, measured in diopters.[22]

Following the method of Green and Campbell,[4] we have drawn straight lines through the data to the left and right of the peaks. In the log linear coordinates, these lines imply that modulation transfer is a decaying exponential function of defocus. On the whole, these lines provide good fits to the data, except for a tendency for the peaks to be flattened at low spatial frequencies. Williams and Boothe[30] found peak flattening at low spatial frequencies in their study of the effects of defocus on contrast sensitivity in monkeys. The dioptric location of the peaks in Figs. 3 and 4 and in data sets not shown did not vary systematically with spatial frequency. Green and Campbell[4] showed that an eye with a dilated pupil became more myopic at low spatial frequencies; its optimal focus shifted. They attributed the effect to primary, undercorrected spherical aberration. However, changes in the location of the peak should be relatively slight at the low spatial frequencies that we studied.[15] Notice that the data in Figs. 3 and 4 show some asymmetries in the tolerance

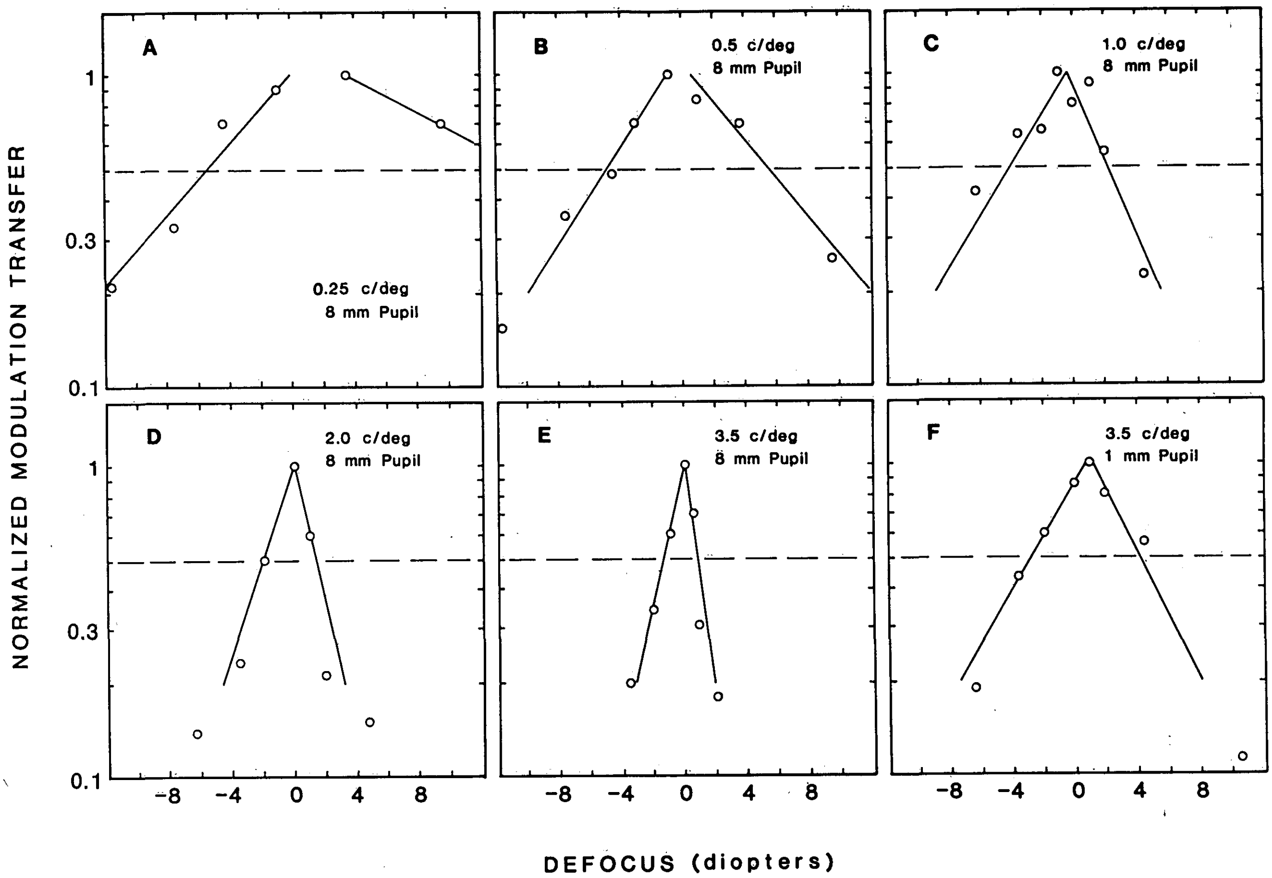


Fig. 3. Modulation transfer as a function of defocus. Each panel shows mean data from one experiment for subject KTM. One point has been omitted from panel F because it falls off the scale to the right. Within each panel, normalized modulation transfer is proportional to contrast sensitivity, with normalization based on maximum contrast sensitivity. Straight lines have been fitted by eye to all the data to the left and right of the peak and are drawn down to an ordinate value of 0.2. Horizontal dashed lines indicate where transfer has dropped to 50% of its peak. We take the breadth of the curves, measured along these half-amplitude lines, as our definition of depth of focus.

to positive and negative defocus. At the lowest frequency, the curves are broader on the positive side, but at the highest frequency, the reverse is found.

In the panels of Figs. 3 and 4, horizontal dashed lines have been drawn to indicate where transfer drops to 50% of its peak. We take the dioptric breadth of the inverted V-shaped curve, measured along these half-amplitude lines, as our definition of depth of focus. For example, in Fig. 3B, KTM's depth of focus is roughly 10 D at 0.5 c/deg.

Notice how the curves in Figs. 3A–3E and 4A–4E grow narrower as the spatial frequency increases. From 0.25 to 3.5 c/deg, KTM's depth of focus decreases from 20.4 to 2.2 D, and GW's depth of focus decreases from 15 to 3.5 D.

Figure 5 shows how depth of focus depends on spatial frequency for subjects with dilated pupils. Each point is the depth of focus estimated from one experiment. Values derived from Figs. 3 and 4 are shown, along with several replications (open symbols). Filled symbols show estimates based on the matching method. There were no systematic differences in estimates of the depth of focus obtained by the two methods, and in no case did comparable mean estimates differ by more than 30%. The X's in Fig. 5 represent values derived from the data of Green and Campbell,[4] and the +'s represent values derived from the data of Charman.[5] Together the data provide estimates of the depth of focus from 0.25 to 30 c/deg. The solid curve shows how depth of focus varies with spatial frequency for an ideal (aberration-free, diffraction-limited) eye with 8-mm-diameter pupil. Below 2 c/deg, empirical estimates of the depth of focus shrink as frequency rises in rough correspondence to the ideal. Above 2 c/deg, the empirical depth of focus does not shrink much further, and it departs increasingly from the ideal.

Optical considerations predict closer adherence to ideal performance for small pupils. The points in Fig. 6 are empirical estimates of the depth of focus derived from experiments with 2-mm-diameter pupils. Indeed, the points remain close to the ideal curve all the way out to 30 c/deg.

Comparison of Fig. 3E with Fig. 3F and Fig. 4E with Fig. 4F illustrate that, for a fixed spatial frequency, depth of focus increases as pupil size decreases. For example, KTM's depth of focus for an 8-mm pupil at 3.5 c/deg is 2.2 D (Fig. 3E), but for a 1-mm pupil it is 6.9 D (Fig. 3F).

Figure 7 shows the role of pupil size more directly. The depth of focus is plotted against the pupil diameter for a fixed spatial frequency of 3.5 c/deg. From 1 to 2.5 mm, the depth of focus decreases in correspondence with the ideal, but for wider pupils it does not decrease much more.

On the whole, Figs. 3–7 indicate that the depth of focus of the human eye is close to values expected from a diffraction-limited, aberration-free system when the pupil diameter is less than about 2.5 mm and the spatial frequency is less than about 2 c/deg. In this domain, the laws of geometrical optics

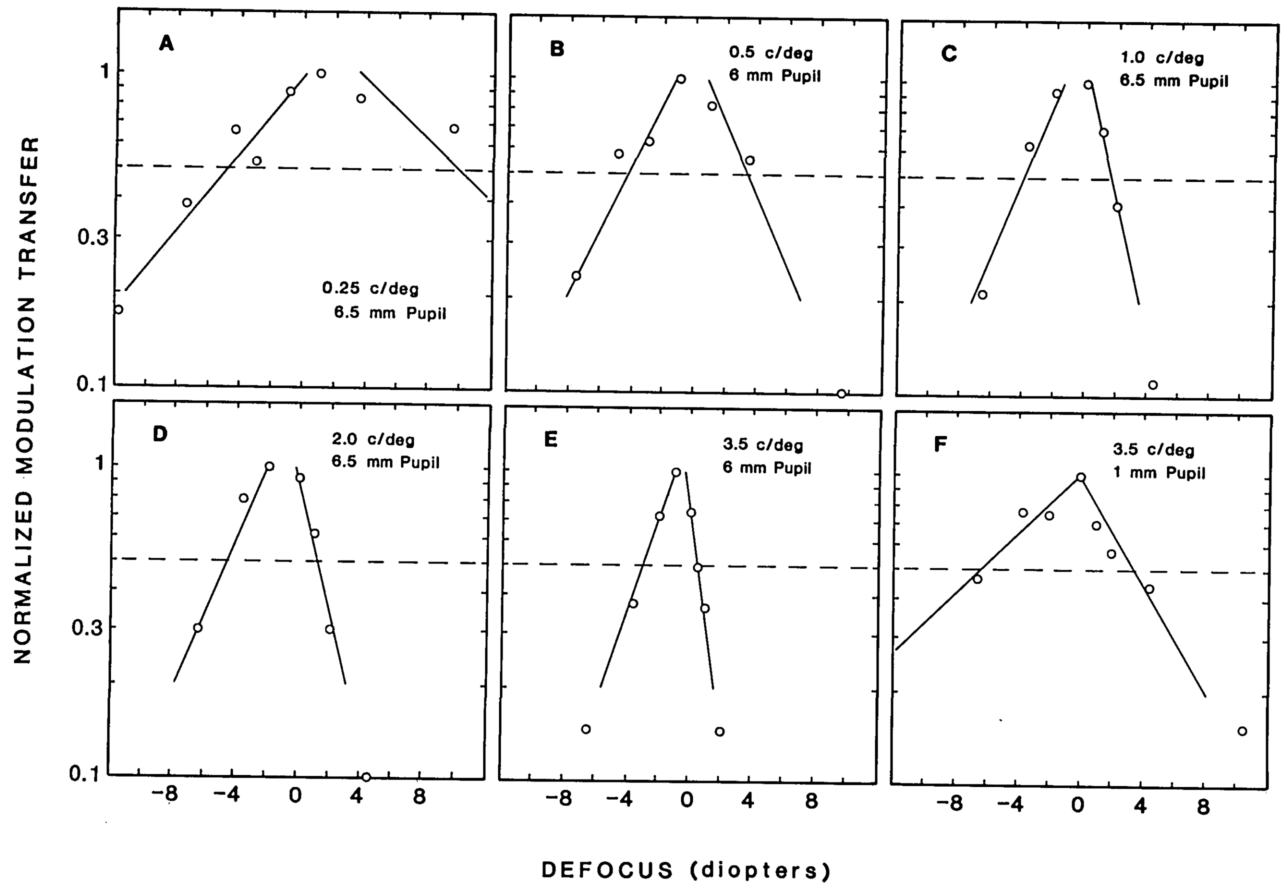


Fig. 4. Modulation transfer as a function of defocus. Details are as in Fig. 3 except that data are for subject GW.

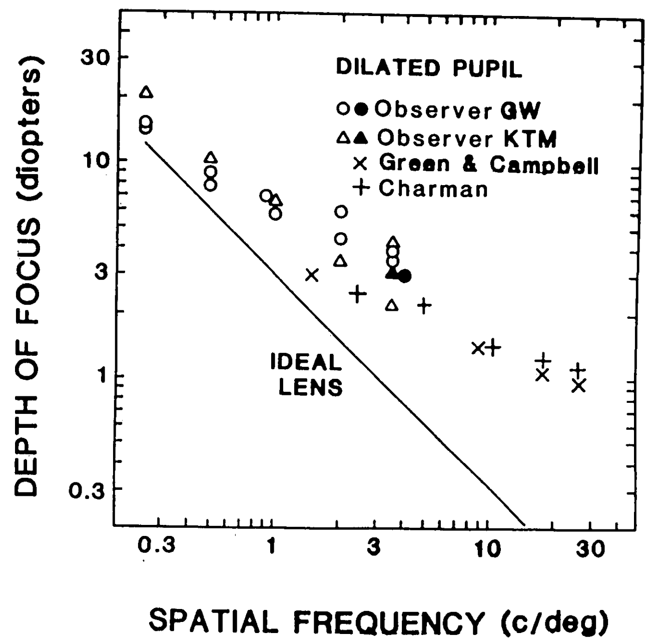


Fig. 5. Depth of focus as a function of spatial frequency for experiments with dilated pupils. Each point is the depth of focus estimated from one experiment. Circles and triangles refer to data from this study, open symbols refer to the threshold method, and filled symbols refer to the matching method. ×'s refer to values derived from the data of Green and Campbell,[4] and +'s refer to the data of Charman.[5] The solid curve labeled ideal lens refers to results to be expected from an aberration-free, diffraction-limited optical system with an 8-mm pupil.

provide a good approximation; depth of focus is inversely proportional to pupil diameter and inversely proportional to spatial frequency.[21,31] These reciprocal relations provide a good first-order approximation of human depth of focus for small pupils and low spatial frequencies. Based on their data for line-spread function in the human eye, Campbell and Gubisch[32] also concluded that the eye is diffraction limited for pupil diameters less than about 2.4 mm.

On the other hand, the depth of focus of the human eye becomes relatively insensitive to pupil size at diameters above about 3 mm and to frequencies above about 4 c/deg. In this domain, the depth of focus lies between about 1 and 3 D, the former corresponding to the widest pupils and highest spatial frequencies.

Other Factors Increasing Tolerance to Defocus

Why does the depth of focus depart from ideal values? We briefly consider five factors.

Chromatic Aberration

The refracting power of the eye is greater for blue light than for red light. As a result, the blue light in a polychromatic target will be imaged closer to the cornea than will the red light. The eye may thereby increase its depth of focus.

Theoretical analysis suggests that the effect of chromatic aberration on the depth of focus is small. We used the method described by van Meeteren[25] to calculate the theoretical MTF for gratings composed of polychromatic light.[33] We calculated the depth of focus for monochromatic and

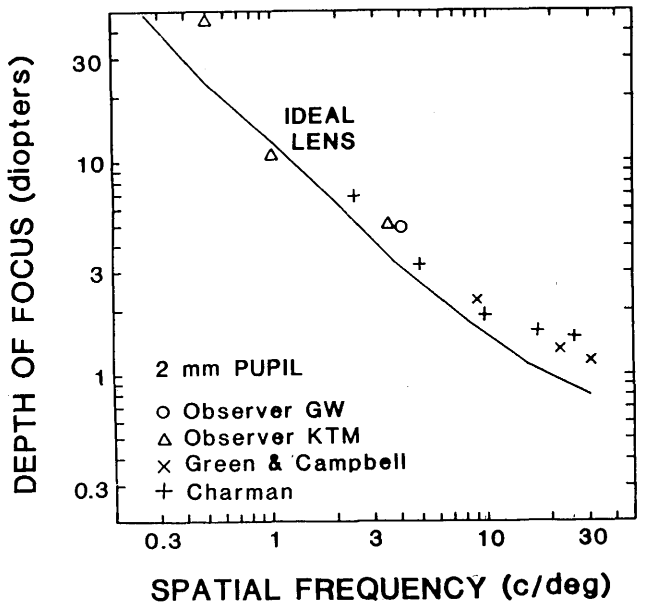


Fig. 6. Depth of focus as a function of spatial frequency for 2-mm-diameter pupils. Details are as in Fig. 5 except that subjects viewed the stimuli through 2-mm-diameter artificial pupils. The ideal lens curve refers to an optical system with a 2-mm pupil.

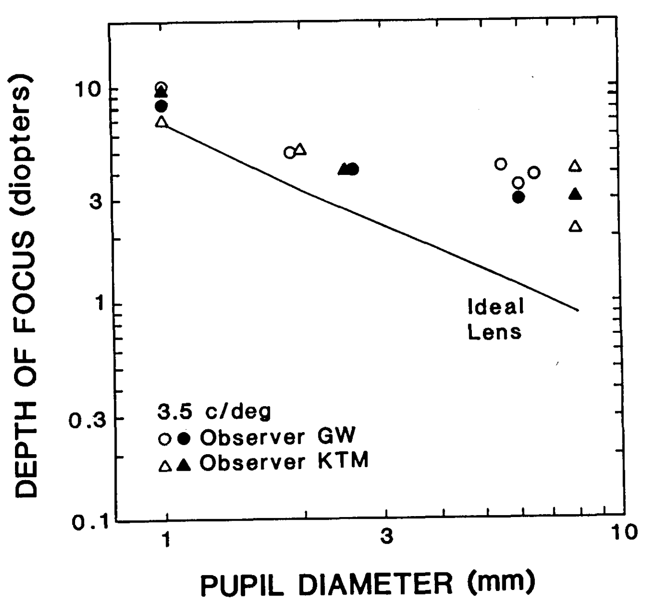


Fig. 7. Depth of focus as a function of pupil diameter for 3.5-c/deg sine-wave gratings.

white grating stimuli at different spatial frequencies and pupil sizes. The theoretical differences caused by color are small. For example, the depth of focus for a monochromatic (575-nm) 1-c/deg grating viewed through a 6.5-mm pupil is only 3% less than that for a white one.

Experimental estimates also reveal only small effects. Campbell[2] found that the depth of focus decreased by only 9% when an achromatizing lens was used in his experiments. He also compared the depth of focus for white light and yellow light (550 nm). The depth of focus was only about 13% smaller for the latter. Campbell and Gubisch[34] compared contrast-sensitivity functions for monochromatic yellow (578-nm) and white gratings. As part of their study, they measured the effect of defocus on the contrast sensitivity for 30-c/deg gratings viewed through a 2.5-mm artificial pupil. Our analysis of their Fig. 5 reveals that the depth of focus was 1.4 times greater for the white grating.

Table 2 reports results of three experiments from which we estimated the depth of focus for monochromatic yellow (577 nm) and polychromatic (white P4 phosphor or desaturated green P31 phosphor) 3.5-c/deg gratings. What little difference we found favored a larger depth of focus for the monochromatic light. Von Bahr[10] alludes to a similar reversal of expectation in a comparison of the depth of focus in sodium yellow light and daylight. Whatever the cause of the small difference that we found, chromatic aberration does not appear to play a major role in increasing tolerance to visual defocus.

Grating Orientation

The optic axis is offset about 5 deg from the fovea toward the optic disk. As pointed out by van Meeteren,[25] optical quality might be better (higher modulation transfer and higher cutoff frequency) and the depth of focus might be less for horizontal gratings than for vertical gratings.

Table 3 reports results of one experiment with GW and repeated experiments with KTM, in which depth of focus was compared for vertical and 3.5-c/deg horizontal gratings. The table also shows spatial-frequency cutoffs for the two subjects.[35] GW shows no orientational differences. However, KTM exhibits a higher cutoff and a smaller depth of focus for horizontal gratings. Her results are consistent with the prediction of better optical quality for horizontal gratings.[36]

Spurious Resolution

Our definition of depth of focus requires us to locate the half-amplitude transfer points. As defocus increases, transfer drops, passing through the half-amplitude point and eventually reaching zero. Further defocus may result in spurious resolution, indicated by a phase reversal in a sine-wave grating image. Still more defocus may result in additional phase reversals separated by points of zero transfer. It is possible that empirical curves like those in Figs. 3 and 4 have been broadened by measurements taken in regions of spurious resolution. There are two reasons why this is unlikely. First, our curves decrease monotonically from the in-focus points. We found no instances of zero transfer or reliable nonmonotonicity. Second, transfer in regions of spurious resolution is generally low,[21] well below the 50% transfer on which we based our definition of depth of focus.

Table 2. Depth of Focus at 3.5 c/deg: Effects of Wavelength Composition

Subject	Pupil Diameter (mm)	Method	Depth of Focus (D)[a] Yellow[b]	White[c]	Ratio (Yellow/White)
GW	6	matching	3.2	3.0	1.07
KTM	1	matching	9.9	9.5	1.04
KTM	1	threshold	9.3	6.9	1.35

[a] Half-amplitude criterion.
[b] Interference filter with λ_{max} = 577 nm.
[c] Matching method, P4 white phosphor; threshold method, P31 desaturated green phosphor.

Table 3. Depth of Focus at 3.5 c/deg: Effect of Grating Orientation

Subject	Pupil Diameter (mm)	Spatial-frequency Cutoff (c/deg)		Depth of focus (D)[a]		Ratio (Vertical/Horizontal)
		Vertical	Horizontal	Vertical	Horizontal	
GW	6.5	38.0	38.0	4.3	4.5	0.96
KTM	8.0	34.2	43.9	4.2	2.1	2.00
KTM[b]	8.0			2.2	1.4	1.57

[a] Half-amplitude criterion.
[b] Repeated measurement with the same conditions.

Nevertheless, because spurious resolution is of some intrinsic interest, apart from its possible effect on depth of focus, we examined it experimentally as described in the Method section.

In Fig. 8, the axes are defocus and spatial frequency. The lines marked FIRST ZERO and SECOND ZERO represent combinations of defocus and spatial frequency associated with zeros in the transfer function of an ideal, defocused lens.[37] The data points represent estimates of the first zero for different amounts of defocus for KTM. Figure 8A shows data for a 2-mm-diameter artificial pupil. Seven of the eight points lie very near the first zero contour, and the eighth point lies near the second zero. These results confirm once again that an eye with a 2-mm pupil is diffraction limited. Figure 8B shows corresponding data for an 8-mm pupil. Here, none of the data lie on the first zero contour, several points lie near the second zero contour, and two points lie still further out. This illustrates once again that an eye with a wide pupil has a greater tolerance to defocus than does an aberration-free, diffraction-limited optical system with the same pupil. The pattern of results shown in Fig. 8B may be related to the Stiles–Crawford effect, as discussed below.

For a specified pupil size and spatial frequency, we can compare the defocus required to produce the first zero in the transfer function (Fig. 8) with the defocus required for half-amplitude transfer (Figs. 3 and 4).[38] This analysis revealed that, in all cases, the half-amplitude defocus was less, with the difference being small for the 8-mm pupil and 3.5 c/deg. These findings confirm that our half-amplitude measures of the depth of focus were not contaminated by spurious resolution.

Spherical Aberration

Spherical aberration is usually considered to be the most important of the monochromatic aberrations, although coma may play an important role as well.[24,39] Like chromatic aberration, spherical aberration acts to increase the depth of focus. Its effect becomes evident only for large pupils (>3 mm; Ref. 25) and for high spatial frequencies (>4 c/deg; Ref. 15).

Charman and Heron[15] present graphs showing how modulation transfer depends on defocus for an eye having 1.5-D undercorrected spherical aberration. Their graphs are based on the calculations of Black and Linfoot.[40] Separate plots are shown for spatial frequencies ranging from 1 to 32 c/deg. Using our half-amplitude definition, we estimated the spatial-frequency dependence of the depth of focus from

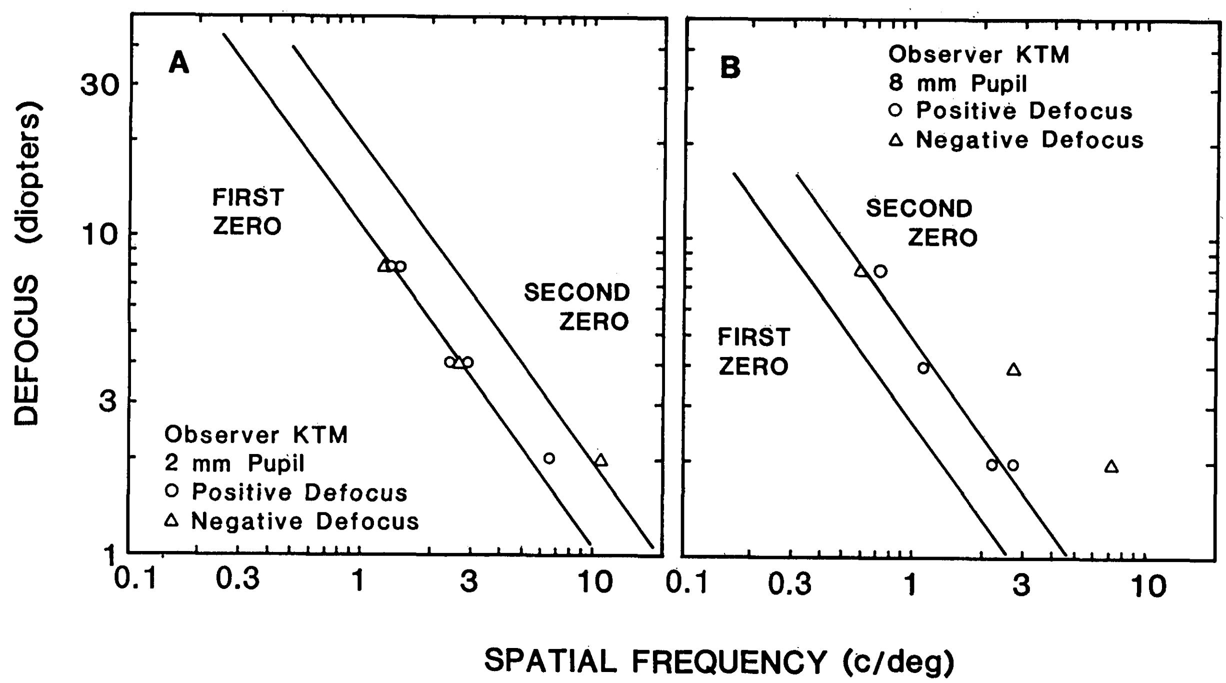


Fig. 8. Spurious resolution in the human eye. The data points show our measurements of the spatial frequency and defocus associated with the first zero in modulation transfer function of KTM's right eye. Circles refer to defocus with positive lenses and triangles to defocus with negative lenses. The lines labeled FIRST ZERO and SECOND ZERO represent combinations of defocus and spatial frequency producing first and second zeros in the transfer functions of an ideal lens. A, 2-mm pupil; B, 8-mm pupil.

their plots. Whereas the depth of focus for an aberration-free lens drops by about a factor of 3.5 from 8 to 32 c/deg (5.4-mm pupil, 555-nm light), the depth of focus for the aberrated system decreased by only a factor of 2. At 32 c/deg, the depth of focus for the aberrated system is about 2.5 times greater than for the aberration-free lens but still at least a factor of 2 below human values.

These considerations suggest that spherical aberration plays an important role in enlarging the depth of focus for moderately wide pupils and moderately high spatial frequencies. Aberrations of the eye, although usually disparaged, have the beneficial consequence of increasing tolerance to defocus.

The Stiles-Crawford Effect

Light rays entering the eye near the edge of a wide pupil are absorbed less efficiently by the photoreceptors than are axial rays. In terms of light absorption, the effective pupil size is reduced compared with the real pupil size: about a 10% reduction for a 4-mm pupil and more for a wider pupil. Campbell[2] has argued that the smaller effective pupil size accounts for the depth of focus of wide pupils being larger than expected on the basis of geometrical optics.

Van Meeteren[25] has a different point of view. He argues that the Stiles–Crawford effect reduces the effectiveness of eccentric rays, thereby limiting the role of off-axis aberrations. Since these aberrations would increase the depth of focus, as in the case of spherical aberration, the Stiles–Crawford effect may actually decrease tolerance to visual defocus.

The Stiles–Crawford effect is optically equivalent to a pupil in which the light transmittance decreases outward from the center. Mino and Okano[41] computed transfer functions for a defocused optical system with such a "shaded" pupil. They compared their results with corresponding transfer functions for a clear pupil. Their theoretical curves demonstrate two points of relevance to the present paper. First, at low spatial frequencies, modulation transfer for the shaded pupil is higher than for the clear pupil, the difference being more pronounced for larger levels of defocus. This means that the shaded pupil has a greater depth of focus and suggests that the Stiles–Crawford effect contributes to the enhanced tolerance to defocus of the human eye at low spatial frequencies. Second, the first zero in the transfer function of a defocused optical system with a shaded pupil occurs at a higher frequency than for the corresponding system with a clear pupil. The shading increases the spatial frequency at which spurious resolution occurs. Such an effect may account for the horizontal displacement of the data in Fig. 8B to spatial frequencies higher than those expected for the first zero of a system with a clear pupil.

In summary, our consideration of the foregoing five factors leads us to conclude that the Stiles–Crawford effect and the monochromatic aberrations, principally spherical aberration, probably play dominant roles in increasing tolerance to defocus. There may be lesser contributions from chromatic aberration and the orientation differences in modulation transfer. Spurious resolution does not play a role.

Relation to Campbell's Definition of Depth of Focus

Our definition of depth of focus refers only to properties of the eye's optics. By comparison, Campbell's blur-detection method for estimating the depth of focus includes effects of neural processing.[2] How can the two be related?

We begin by showing how to derive depth-of-focus estimates from our transfer data for criteria other than the half-amplitude criterion. From Figs. 3 and 4 we estimated the width of the curves along the horizontal lines at ordinate values of 0.5. Had we chosen a higher criterion, e.g., reduction to just 90% of peak transfer, the estimate of the depth of focus would have been much smaller. Suppose that decaying exponentials represent the relationship between normalized modulation transfer T and dioptric defocus D (straight lines in Figs. 3 and 4). We have

$$T = \exp(-kD),$$

where k is a decay constant. Let the defocus associated with 50% normalized transfer be $D_{0.5}$. Solving for k and replacing it in our equation, we have

$$T = \exp[-(0.7/D_{0.5})D].$$

Suppose now that we want the defocus D_a associated with some arbitrary value of normalized transfer T_a. We solve the preceding equation to obtain

$$D_a = (-D_{0.5}/0.7)\ln(T_a).$$

For example, if $T_a = 0.9$ (i.e., 90% criterion), $D_a = 0.15D_{0.5}$. In other words, the defocus required to reduce transfer to 90% of the peak value is just 15% of the defocus required to reduce transfer to 50% of is peak value.

The image of a defocused sine-wave grating is a sine-wave grating of the same spatial frequency but lower contrast. Studies of contrast discrimination[42] indicate that high-contrast gratings can be discriminated if they differ in contrast by about 10%. For such gratings we would expect that the amount of defocus required to reduce transfer to 90% of its peak would be the amount of defocus that produces a just-noticeable difference in contrast. From our measurements of normalized transfer, we know that roughly 1.5 D of defocus, positive or negative, is required at 3.5 c/deg to reduce transfer to 50% of its peak for a dilated pupil. The example at the end of the previous paragraph indicates that only 15% of this, about 0.22 D, is required to reduce transfer to 90% of maximum, the just-noticeable reduction. Campbell,[2] using a 6-mm pupil, found that the just-noticeable defocus for his high-contrast disks was 0.21 D. This congruence of results encourages us to speculate that Campbell's subjects based their blur-detection judgments on the output of spatial-frequency channels tuned to about 4 c/deg, close to the peak of the contrast-sensitivity function.

Campbell[2] also examined the effect of target contrast on the depth of focus. From his Fig. 3, the depth of focus increased by a factor of 1.9 when contrast decreased by a factor of 5. A fivefold reduction in target contrast results in a rise by a factor of about 2 in the contrast Weber fraction,[42] from about 10 to 20%. From the preceding equations, about 2.1 times more defocus is required to reduce transfer to 80% of maximum than to reduce it to 90% of maximum. We therefore predict that a fivefold reduction in target contrast will result in an increase in the depth of focus by a factor of 2.1. This is very close to the factor of 1.9 measured by Campbell.

In this subsection we have linked an optical definition for the depth of focus of the eye with Campbell's blur-detection definition. We have done so by using psychophysical data from contrast discrimination.[43] In addition, we have found

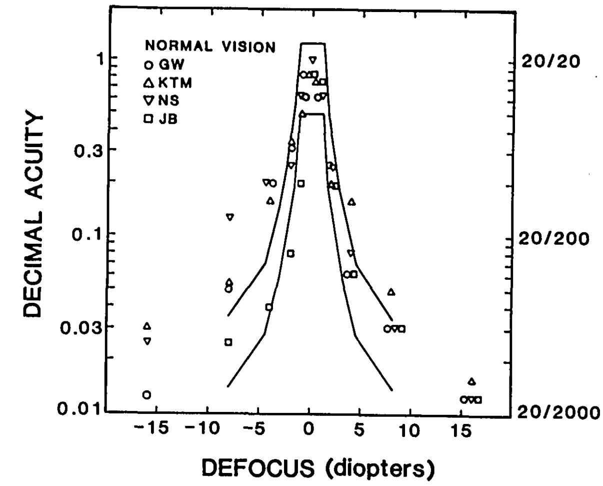


Fig. 9. Acuity is plotted as a function of defocus for four normal subjects (see Table 1). The solid lines are predictions based on our modulation-transfer data and spatial-frequency bandwidth considerations (see the text).

a plausible explanation for the contrast dependence of the depth of focus found by Campbell.[2]

Letter Acuity as a Function of Defocus

We measured letter acuity as a function of defocus for four normal subjects. Their pupils were dilated, and their accommodation was paralyzed. Individual results are shown in Fig. 9. The right ordinate gives acuity in Snellen notation, and the left ordinate gives its decimal equivalent. The solid lines will be discussed below.

The relation between decimal acuity A and dioptric defocus D can be expressed by two power laws (least-squares best fits in log–log coordinates): for positive defocus, $A = 0.64/D^{1.45}$; for negative defocus, $A = 0.51/D^{1.13}$. Although acuity declines with defocus a little more rapidly than inverse proportionality, a rule of thumb is that $A \sim 0.5/D$. Our findings are quite similar to those reported by Tucker and Charman.[9] Corresponding power law exponents derived from data in their Figs. 4 and 5 are 1.28 and 1.7 for positive and negative defocus, respectively.

Figure 9 shows how much defocus can be tolerated when the task is recognition of characters of a given size. For example, if the task is to read 20 of 100 letters (subtending 25 arcmin), up to 2 D of defocus can be tolerated. Tucker and Charman[9] defined the total depth of focus for a given acuity level as the lateral separation between positive and negative data points at specified ordinate values in plots like Fig. 9. Figure 10 shows the total depth of focus as a function of acuity for the mean data in Fig. 9 and for data from Tucker and Charman[9] (their Fig. 6, data for a 7.5-mm pupil). In the overlap region, the two sets of data are in good agreement. For a decimal acuity of 2.0 (Snellen 20/10), the total depth of focus is 0.4 D, that is, ±0.2 D. This accords well with the precision achievable in clinical refraction. Notice that for a decimal acuity of 0.1 (Snellen 20/200), the total depth of focus is more than 7 D. This means that a subject with normal vision can just read the line on the letter chart defining legal blindness, the 20/200 line, when defocused by ±3.5 D.

How can we relate these acuity measures to our modulation-transfer estimates of the depth of focus? A connection is possible through the intermediary of spatial-frequency filtering. Defocus may be crudely regarded as a kind of low-pass filtering because, for the most part, high frequencies are attenuated more than low frequencies.[44] If we can determine how low-pass spatial-frequency filtering affects letter identification, we can use our modulation-transfer data to estimate the effects of defocus on acuity.

Legge *et al.*[16] showed how low-pass filtering affects reading. They measured reading rate as a function of filter bandwidth. The low-pass filtering was done with a calibrated ground-glass diffuser. Filter bandwidth may be defined as the half-amplitude spatial frequency of the diffuser's MTF. Legge *et al.* reduced the bandwidth until a critical bandwidth was reached, at which reading performance began to deteriorate. The dashed line in Fig. 11, labeled READING, summarizes how the critical bandwidth depended on the character size.[45] Not surprisingly, the critical bandwidth increases as the character size diminishes: higher spatial frequencies are required in order to read smaller letters. Rubin and Siegel[17] used similar methods to measure critical bandwidths for the recognition of individual letters. The subject's task was to indicate which one of ten test letters was presented on the screen. The blur was increased until recognition fell below a criterion level. The line labeled LETTER RECOGNITION in Fig. 11 summarizes how critical bandwidths depended on the character size in this experiment. The bandwidths for letter recognition are substantially lower than those for reading. This may be due to the simpler nature of the task: identification of one static letter from a target set of ten rather than fast reading of novel text.[46] We take the letter-recognition results to form a lower bound and the reading results to form an upper bound on the bandwidth requirements for a clinical acuity task.

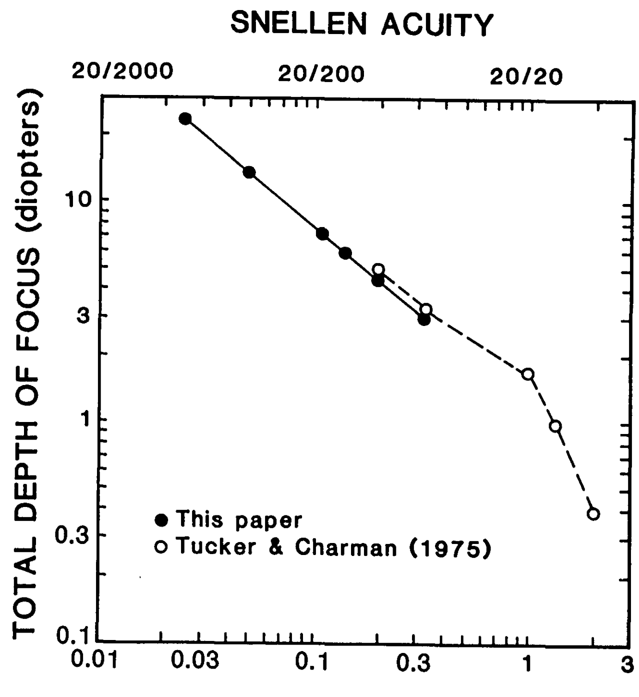


Fig. 10. Total depth of focus, as defined by Tucker and Charman,[9] is plotted as a function of acuity (see the text).

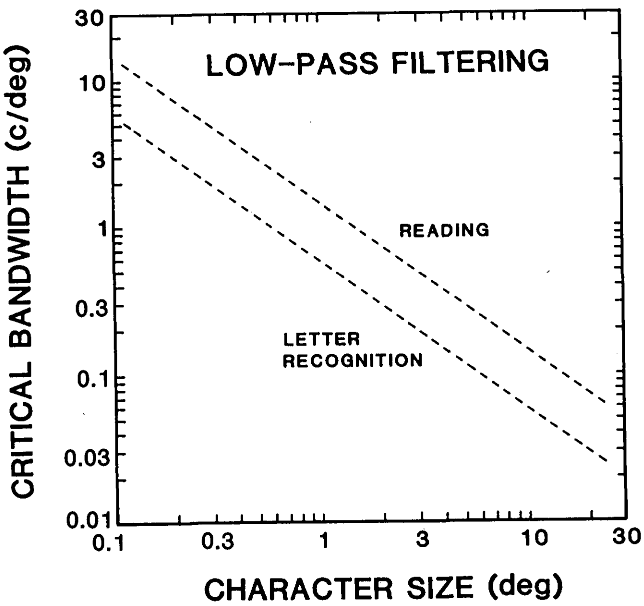


Fig. 11. Critical bandwidths for reading and letter recognition. The dashed curves summarize the findings of experiments in which the minimum spatial-frequency bandwidths for reading[16] and for letter recognition[17] were measured as a function of character size. These bandwidths can be used in relating our modulation-transfer definition of depth of focus to the effects of defocus on acuity.

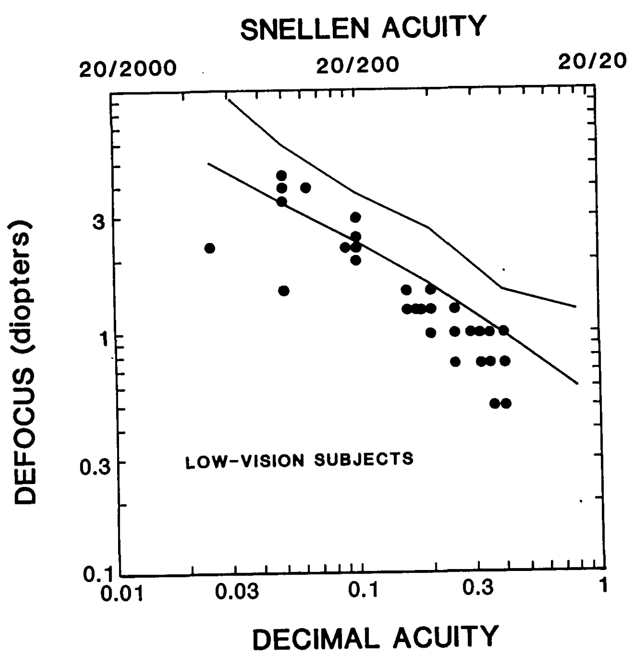


Fig. 12. Tolerance to defocus in low vision. Each point refers to one low-vision eye. The positive-lens defocus required to degrade acuity by one line on a chart (0.1 log unit in character size) is plotted as a function of the subject's acuity. The solid lines are based on predictions from our modulation-transfer data and spatial-frequency filtering considerations.

With the bandwidth requirements of the acuity task in hand, we may now proceed to predict the effects of defocus. Consider characters of a given size, say, 1 deg. From Fig. 11, we estimate that a bandwidth in the range 0.6–1.5 c/deg would be required to read the 1-deg characters on the acuity chart. From our data in Fig. 5, the depth of focus at 1.5 c/deg is about 5.5 D, and that at 0.6 c/deg is about 8 D. Half of these values, 2.75 and 4 D, represent the positive or negative lens defocus that reduces transfer to half amplitude. According to our analysis, defocus in the range 2.74–4 D will reduce acuity so that the smallest letters that can be read subtend 1 deg (corresponding to a decimal acuity of 0.083 and a Snellen acuity of 20/240). This calculation was carried out for a range of character sizes to produce the pairs of solid lines in Fig. 9, the outer pair corresponding to the letter-recognition bandwidths and the inner pair corresponding to the reading bandwidths. Overall, the data in Fig. 9 cluster around the predictions fairly well. As might be anticipated, there is a somewhat better fit to the predictions from the letter-recognition bandwidths (outer pair of lines).

In summary, we have considered defocus as a kind of low-pass spatial-frequency filtering. Using independent data on the spatial-frequency bandwidth requirements of reading and letter recognition, we have predicted how defocus should affect acuity. In doing so, we have linked our modulation-transfer analysis of defocus with our acuity results.

Tolerance to Defocus in Low Vision

We examined tolerance to defocus in 30 low-vision eyes. Subjects having a variety of pathologies and degrees of vision loss were tested. Our aim was to determine whether, as predicted, decreased acuity was associated with increased tolerance to defocus. Standard clinical methods were used to obtain the best refraction. Letter acuity was measured, and then the eye was defocused with positive lenses until acuity decreased by one line on the chart (a change in character size by 0.1 log unit). This positive-lens power was taken as the subject's tolerance to defocus.

In Fig. 12, tolerance to defocus is plotted against acuity. Each point in the scatter diagram refers to one low-vision eye. As predicted, tolerance increases as acuity decreases: the correlation coefficient based on log values is −0.87. Tolerance ranges from about 0.5 D for subjects with nearly normal acuity to about 5 D for subjects with a decimal acuity of 0.025 (Snellen 20/800).

The solid lines in Fig. 12 are predictions derived in the same way as those in the previous section. They are based on our modulation-transfer results coupled with spatial-frequency bandwidth requirements for reading (lower solid line) and letter recognition (upper solid line). The data cluster more closely around the lower solid line.

We draw two conclusions from the low-vision experiment. First, tolerance to defocus increases as acuity decreases. Although it is important to refract low-vision subjects carefully to optimize their residual vision, the accuracy of subjective refraction becomes more and more crude for lower and lower acuities. Put more positively, individuals with low visual acuity benefit from an enhanced depth of focus. The second conclusion is that tolerance to defocus in low vision can be understood, at least in part, from two properties of normal vision: the depth of focus at low spatial frequencies and the spatial-frequency bandwidths of letter recognition and reading.

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$$P_e = (P - D')/[1 - d(P - D')].$$
The corresponding magnification M is
$$M = (dD' + 1)/[1 - d(P - D')].$$
In the figures, defocus refers to values of P_e. Values of spatial frequency cited in the Results section reflect a correction for lens magnification.
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33. According to this method, modulation transfer is calculated for wavelengths taken in small steps across the visible spectrum, taking into account the different degrees of defocus resulting from chromatic aberration. A weighted superposition of the values is taken, with the weights determined by the target luminance at each wavelength. To perform this calculation for gratings displayed on a P31 phosphor, we used the radiometric spectral data compiled by Bell.[48]
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35. The high-frequency cutoffs were measured with slightly different conditions for the two observers. Intersubject comparisons are not meaningful, but horizontal-versus-vertical comparisons for a given subject are.
36. Inequality in the spherical correction for horizontal and vertical axes (astigmatism) would not produce this pattern of results. Astigmatism would affect the location of the peaks of curves like those in Figs. 3 and 4 but would not affect the breadth of the curves.
37. According to geometrical optics, there is a reciprocal relationship between spatial frequency and defocus in the specification of zeros of the transfer function. From Ref. 21, the spatial frequency f at which the first zero occurs is $f = 21.19/PD$, where P is pupil diameter in millimeters and D is defocus in diopters. At low spatial frequencies or for large defocus, values predicted from wave optics are quite similar.
38. For this comparison, we took the first zero contour as the location for data points in Fig. 8A and the second zero contour for data points in Fig. 8B.
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43. It might be argued that small errors in focus produce increasingly large contrast decrements at higher spatial frequencies.[25] Hence observers should rely on high spatial frequencies for blur detection, not on the output of a 4-c/deg channel. Although this argument may be true for aberration-free optical systems, the data of Fig. 5 indicate that the human eye has a relatively constant depth of focus at moderate and high spatial frequencies. Moreover, the high-frequency decline in contrast sensitivity combined with the rolloff in the amplitude spectra of most real-world objects probably makes blur detection based on high spatial frequencies unreliable.
44. Spurious resolution complicates this analogy. Moreover, for an ideal lens, the high-frequency cutoff, beyond which there is no further transfer, positive or negative, is unaffected by defocus.
45. Legge *et al.*[16] used a $1/e$ definition rather than a half-amplitude definition of filter bandwidth. They referred to bandwidths in units of cycles per character rather than cycles per degree. If the former is divided by character size in degrees, it gives the latter.
46. According to Fig. 11, critical bandwidths for 0.1-deg characters are 6 c/deg (letter recognition) and 15 c/deg (reading). These bandwidths correspond to just 0.6 and 1.5 cycles per character, respectively. Perhaps the half-amplitude definition gives misleadingly low estimates because information passed by the ground-glass diffuser at spatial frequencies higher than the half-amplitude frequency may be used by subjects in performing the tasks.
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